Nonparametric estimation for fractional Black-Scholes processes with random effects

Ichi Souad and El Maroufy Hamid

Abstract We present the problem of estimating the density from Stochastic Differential Equations with drift depending on random effects driven by normalized fractional Brownian motion. We extend the existing works, which considered the problem of stochastic differential equations with random effects driven by fractional Brownian motion, but the Hurst parameter is considered known on the range $(1/2, 1)$. In this communication, we propose a nonparametric adaptive strategy to estimate the density of random effects from the Black-Scholes model driven by normalized fractional Brownian motion. More precisely, we build estimators of density based on deconvolution tools that estimators depend on two tuning parameters which were selected in a data-driven way and study their mean integrated squared error when Hurst parameter H belongs to $(0, 1)$ and the number of subjects N tends to infinity.

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Ichi Souad
Laboratory LMA, FST of Beni Mellal, Sultan Moulay Slimane University, Morocco, e-mail: ichi.souad@gmail.com

El Maroufy Hamid
Laboratory LMA, FST of Beni Mellal, Sultan Moulay Slimane University, Morocco, e-mail: h.elmaroufy@usms.ma

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