Mean curvature and mean shape for multivariate functional data under Frenet-Serret framework

Juhyun Park[1,2], Nicolas Brunel[1,2,3], Perrine Chassat [2]

[1] ENSIIE
[2] Université Paris Saclay, CNRS, Laboratoire de Mathématiques et Modélisation d'Evry
[3] Quantmetry

Abstract: The analysis of curves has been routinely dealt with using tools from functional data analysis, mostly for one-dimensional curves and increasingly for multi-dimensional curves. A standard method to handle these functional data relies on the efficient linear representation of the curves, often through the representation of functional principal component analysis and its variations. However, its extension to multi-dimensional curves poses a new challenge due to its inherent nonlinear features that are difficult to capture with the classical approaches that rely on linear approximations. In this work, we focus on the analysis of three-dimensional spatial curves and propose an alternative notion of mean that reflects shape variation of the curves. Based on a geometric representation of the curves through the Frenet-Serret ordinary differential equations, we introduce a new definition of mean curvature and mean torsion, which form the basis of the mean shape corresponding to the mean ordinary differential equation. This new formulation of the mean for multi-dimensional curves allows us to integrate the parameters for the shape features into the unified functional data modelling framework. In addition, it can be interpreted as a generalization of the elastic mean of the curves based on the square root velocity function representation. We formulate the estimation problem of the functional parameters in a penalized regression framework and develop an efficient algorithm based on the approximation of the solution paths to the underlying differential equation model. We demonstrate our approach with both simulated data and real data examples and provide comparison with existing methods.

Keywords:
functional data; multidimensional curves; curvature; shape analysis