# BEAUTY Powered BEAST

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**Abstract**

We study inference about the uniform distribution with the novel **b**inary **e**xpansion **a**pproximation of **u**niformi**ty** (BEAUTY) approach. Through an extension of the celebrated Euler's formula, we approximate the characteristic function of any copula distribution with a linear combination of means of binary interactions from marginal binary expansions. This novel characterization enables a unification of many important existing tests through an approximation from some quadratic form of symmetry statistics, where the deterministic weight matrix characterizes the power properties of each test. To achieve a uniformly high power, we study test statistics with data-adaptive weights through an oracle approach, referred to as the **b**inary **e**xpansion **a**daptive **s**ymmetry **t**est (BEAST). By utilizing the properties of the binary expansion filtration, we show that the Neyman-Pearson test of uniformity can be approximated by an oracle weighted sum of symmetry statistics. The BEAST with this oracle leads all existing tests we considered in empirical power against all complex forms of alternatives. By approximating this oracle with data-adaptive weights, we develop the BEAST that improves the empirical power of many existing tests against a wide spectrum of common alternatives while providing clear interpretation of the form of non-uniformity upon rejection.

*Keywords:* Nonparametric Inference; Test of Uniformity; Characteristic Function; Binary Expansion; Euler's Formula