Detecting the direction of (hyper)spherical signals

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Abstract. We consider one of the most important problems in directional statistics, namely the spherical location testing problem, whose null is that the modal location $\boldsymbol{\theta}$ of a Fisher–von Mises–Langevin (FvML) distribution on the p-dimensional unit sphere $\mathcal{S}^{p-1}$ coincides with a given location $\boldsymbol{\theta}_0$. The underlying concentration parameter κ plays the role of a nuisance. We derive local asymptotic normality (LAN) results in a general high-dimensional framework where the dimension $p = p_n$ goes to infinity, at an arbitrary rate, with the sample size n, and where the concentration κ_n behaves in a completely free way with n, which offers a spectrum of problems ranging from arbitrarily easy to arbitrarily challenging ones. We identify seven asymptotic regimes, depending on the convergence/divergence properties of (κ_n), that yield different limiting experiments and different contiguity rates. In each regime, we derive Le Cam optimal tests and we compute, from the Le Cam third lemma, asymptotic powers of the classical Watson test under contiguous alternatives. To obtain a full understanding of the non-null behavior of this test, we derive its local asymptotic powers in the broader, semiparametric, model of rotationally symmetric distributions. Monte Carlo studies show that finite-sample behaviours remarkably agree with our asymptotic results.