Inference in Controlled Branching Processes: ABC-SMC methodology

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Abstract:
Controlled branching processes (CBPs) are stochastic growth population models in which the number of individuals with reproductive capacity in each generation is determined by random control functions. The behaviour of these processes is strongly related to their main parameters, corresponding to the offspring and control distributions. In practice, these parameters are unknown and their estimation is necessary.

In this work we deal with the estimation of the posterior distribution of the parameters of interest for CBPs without explicit likelihood calculations. We consider the sample given by the population sizes at every generation and the number of progenitors of the last generation, and we focus on the case where we have no prior knowledge of the maximum number of offspring that an individual can produce. Our approach has two steps. In the first stage, we estimate the posterior distribution of the maximum progeny per individual using an approximate Bayesian computation (ABC) algorithm for model choice with the raw data and based on sequential importance sampling, namely ABC SMC. In the second step, using the values simulated in the previous stage, we estimate the posterior distribution of the main parameters of a CBP by applying the rejection ABC algorithm with an appropriate summary statistic and a post-processing adjustment. We show the accuracy of the proposed methodology via simulated examples and via real data from models that incorporates a carrying capacity, in both cases making use of the statistical software R.

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