We construct a generalization of the Ornstein-Uhlenbeck processes on the cone of covariance matrices endowed with the Log-Euclidean and the Affine-Invariant metrics. Our development exploits the Riemannian geometric structure of symmetric positive definite matrices viewed as a differential manifold. We then provide Bayesian inference for discretely observed diffusion processes of covariance matrices based on an MCMC algorithm built with the help of a novel diffusion bridge sampler accounting for the geometric structure. Our proposed algorithm is illustrated with a real data financial application.