Elastic depths for detecting shape anomalies in functional data

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Abstract

We propose a new family of depth measures called the elastic depths that can be used to greatly improve shape anomaly detection in functional data. Shape anomalies are functions that have considerably different geometric forms or features from the rest of the data. Identifying them is generally more difficult than identifying magnitude anomalies because shape anomalies are often not distinguishable from the bulk of the data with visualization methods. The proposed elastic depths use the recently developed elastic distances to directly measure the centrality of functions in the amplitude and phase spaces. Measuring shape outlyingness in these spaces provides a rigorous quantification of shape, which gives the elastic depths a strong theoretical and practical advantage over other methods in detecting shape anomalies. A simple boxplot and thresholding method is introduced to identify shape anomalies using the elastic depths. We assess the elastic depth's detection skill on simulated shape outlier scenarios and compare them against popular shape anomaly detectors. Finally, we use hurricane trajectories to demonstrate the elastic depth methodology on manifold valued functional data. Supplementary materials, including additional simulations, data examples, and an R-package are available online.

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