Analysis of daily precipitations from synoptic stations using non-homogeneous hidden Markov model

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Abstract

In this work, we used a non-homogeneous hidden Markov model with a special structure capturing the semi-property of hidden semi-Markov approach, to make comparative analyses of different daily precipitations data from various cities. From the estimated parameters by applied the model for these different daily precipitations, we analyzed the level of the precipitations, the regularity of the rainy seasons, and the impact of climate change in each station. the model allowed us to show which city is more impacted by climate change and which one has good rainy periods.

Keywords: hidden Markov model, climate change, daily precipitations.

1 Introduction

About its latitude position (between 6°30 and 12°30 of North latitude), Benin is among countries in which the climate is hot and wet in the intertropical zone. There, the temperatures are constantly high, and by the way, Benin has 25°C as the mean for the whole country. From April to November in the south and from June to October in the north, blows a wet wind from the ocean: it is maritime trade wind or monsoon. From November to the beginning of May in the north and December to march, in the south blows a dry wind coming from the Sahara: it is the continental trade wind. These two masses of air (monsoon and continental trade wind) repulse themselves alternately northwards and southwards. Their zone of contact is called Forehead Intertropical (F.I.T.) or Forehead of monsoon in our region. It is the main seat of all the atmospheric perturbations which induce the precipitations.

Due to Benin lengthening in latitude, the rainy season is progressively observed from the south to the north. Also, the dry season starts earlier and lasts more in the north than in the south. Thus, there are many climate nuances in Benin. In the south zone which is from Cotonou latitude to Savè latitude, we have four seasons, a great rainy season from April to July, a small dry season from August to September, a small rainy season from October to November, and finally a great dry season from December to march. Meanwhile, in the north, we have two seasons. It means one dry season from November to the beginning of May and one rainy season from May to October.

The dearness who lived in Benin in the 1970s was one of the most important and continuous by its rigor, its extension, and its persistence. It is the main cause of rainfall disturbances. Sev-eral authors have looked into the analysis of physics perturbations, which lead to an abnormal distribution of precipitation in time and space. Some like [5]; [20]; [15], and [19] situate the problem in the general framework of natural phenomena of climatic fluctuation. Others like [9] consider that the current drought of the Sahel is an aperiodic phenomenon of the current transition between a stable state of precipitation "quasi-normal" and a stable state of "sub-normal" precipitation. Others like [11], [12],[4] and [10] consider that the inter-annual variability of

precipitation in Africa is related to the variation of the temperatures of surfaces oceanic in the Gulf of Guinea or the variation of the concentration of water vapor in the atmosphere and humidity of the ground, as well as the variation of the force, the position, and the placement in time, of the intertropical convergence zone. But whatever the proposed explanation, all the authors underline the extreme and prolonged character of this phenomenon and its impact on the economic and social development of the region. They generally situate the beginning of this seriousness towards the years 1968-1969 [18] a paroxysm towards years 1971-1972, and a new primer at the beginning of the 1980s [16]. In Benin this period is around the 1970s [2].
To remedy the negative consequences of its disturbances, it is important to analyze the daily precipitation, in the sense of the evolution of dry sequences, precipitations maximum and the number of rainy days. For this analysis a lot of models the statistics have been worked out. Afouda [1] analyzing the daily precipitation in the posts pluviométriques of the region has highlighted the Markovian character of the distribution of dry sequences during the year. [2] have shown that the physical phenomena which are the origin of Annual precipitations in Benin and West Africa lead to a modelization of dry (or wet) sequences by the 1st order Markov chain; this study enabled us to identify the increase in the frequency of tax credits in the sub-period of 1973 – 1993 compared to the sub-period of 1952 – 1972.
This paper is organized as follows. In section 2, we presented the data and the model. In section 3, numerical investigation of the results is given. In section 4, comparative analysis of stations is presented. Concluding remarks are incorporated in section 5.

2 Materiels

In this section, we briefly presented the data and the model.

For the four stations, the data of precipitations are collected during the time of the 01/01/1921 to the 31/12/2007, that is to say 31755 days. The following table summarizes the geographical coordinates and the rise of each of the cities where stations, the start, andin the cities where the stations are

Station	Latitude	Longitude	Rise(en m)	% wet days
Savè	08^o02’	02^o28’	199.0	26%
Parakou	09^o21’	02^o36’	392.0	24%
Kandi	11^o08’	02^o56’	290.0	22 %
Natitingou	10^o19’	01^o23’	460.0	29%

Table 1: Presentation of the synoptic stations

The non-homogeneous hidden Markov model that we used for the analysis of the precipitation data is detailed in the papers [7] and [8], here we used it in the particular case when the set of states $E = \{1,2\}$, where the states 1 and 2 are favorable respectively to dry days and wet days. In this case, the transition probabilities are :

$$\Pi_{11}(t) = \frac{\exp(a_1 \cos(\frac{2\pi t}{365} + \phi_1))}{\exp(a_1 \cos(\frac{2\pi t}{365} + \phi_1)) + 1}, \ \Pi_{12}(t) = 1 - \Pi_{11}(t),$$

$$\Pi_{22}(t) = \frac{\exp(a_2 \cos(\frac{2\pi t}{365} + \phi_2))}{\exp(a_2 \cos(\frac{2\pi t}{365} + \phi_2)) + 1}, \text{ and } \Pi_{21}(t) = 1 - \Pi_{22}(t),$$

and the density function is defined as follows :

$$f_1(y_t) = f(Y_t = y_t|X_t = 1) = \begin{cases} \pi_1 & \text{if } y_t = 0 \\ (1-\pi_1)(\frac{1}{\sqrt{2\pi}\sigma_1 y_t} \exp(-\frac{1}{2}(\frac{\ln(y_t)-\mu_1}{\sigma_1})^2)) & \text{if } y_t > 0 \end{cases}$$

$$f_2(y_t) = f(Y_t = y_t|X_t = 2) = \begin{cases} \pi_2 & \text{if } y_t = 0 \\ (1-\pi_2)(\frac{1}{\sqrt{2\pi}\sigma_2 y_t} \exp(-\frac{1}{2}(\frac{\ln(y_t)-\mu_2}{\sigma_2})^2)) & \text{if } y_t > 0. \end{cases}$$

with a_1 the amplitude relative to the transition from state 1 to state 1, a_2 the amplitude relative to the transition from state 2 to state 2, the phase ϕ_1 allows the beginning of the period of state 1 and the phase ϕ_2 allows the beginning of the period of state 2.

3 Results

To apply the model to data, we take the model complexity $K = 1$ and $E = \{1,2\}$, with 1 the state favorable for dry days (dry season), and 2 the state favorable for wet days (the rainy season). So we have 8 parameters to be estimated: the parameters a_{11} , a_{22} , μ_1 , μ_2 , σ_1, σ_2, π_1, and π_2. To apply the model to observed data, we choose the daily precipitation data registered in Synoptics stations of four cities: Kandi, Parakou, Savé, and Natitingou; in the period between January 1921 and December 2007. To take into account the evolution of the parameters (analyzing the evolution of the precipitation), we will estimate them on sub-data of different periods (periods of 2 years, 5 years, 10 years, and 20 years).
The quality of precipitation during the rainy season will be strongly explained by the shape and scale parameters of state 2, the others parameters will allow supporting the analysis.

3.1 Application to data of Kandi

Fitted the model to each of the 43 sub-data of two-year periods, 17 sub-data of five-year periods, 8 sub-data of ten-year periods, and 4 sub-data of twenty-year periods, we observed :

- at period 25 (1970 – 1971) of two-year periods, 8 (1966 – 1971) of five-years periods (figure 1), 4 (1961 – 1970) of ten-year periods, 2 (1960 – 1980) of twenty-year periods, the simultaneous reduction of the amplitudes, this mean the installation of dry period in rainy seasons, which impact the precipitation;

- the curves of the evolution of the shape parameters μ_2 (for two-year, five-year, ten-year, twenty-years periods) decrease, while the curves of the evolution of scale parameters σ_2 increase, proves that the rainfall decreases at Kandi concerning time and a gap growth between the amount of precipitation from one day to another, more simply the level of bad quality of the rainy seasons' increases concerning time. Moreover, after period 20 (1961 – 1962) of two-year periods, 8 (1961 – 1966) of five-year periods, 4 (1961 – 1970) of ten-year periods (figure 2) and 2 (1961 – 1980) of twenty-year periods, the values of the shape parameters (μ_2) is less than the values of scale parameters (σ_2), this mean that after 1961, we have the succession of bad quality of rainy season in Kandi.

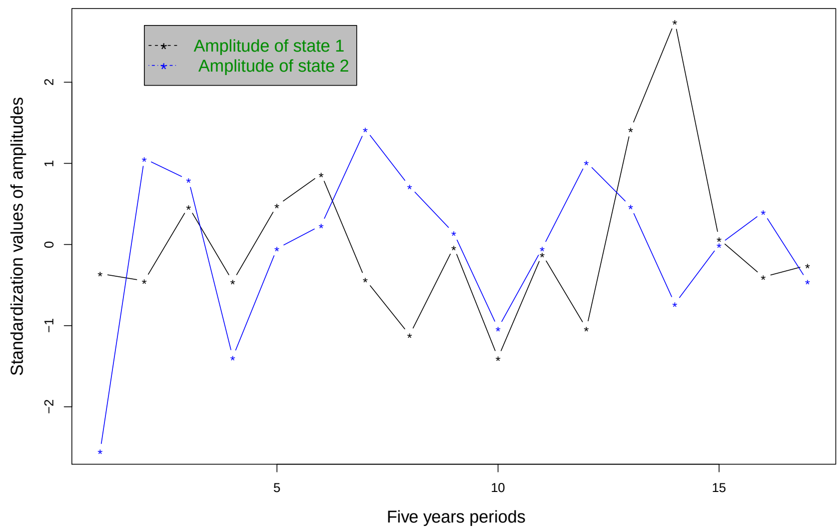

Figure 1: Evolution of amplitudes parameters on five years periods at Kandi

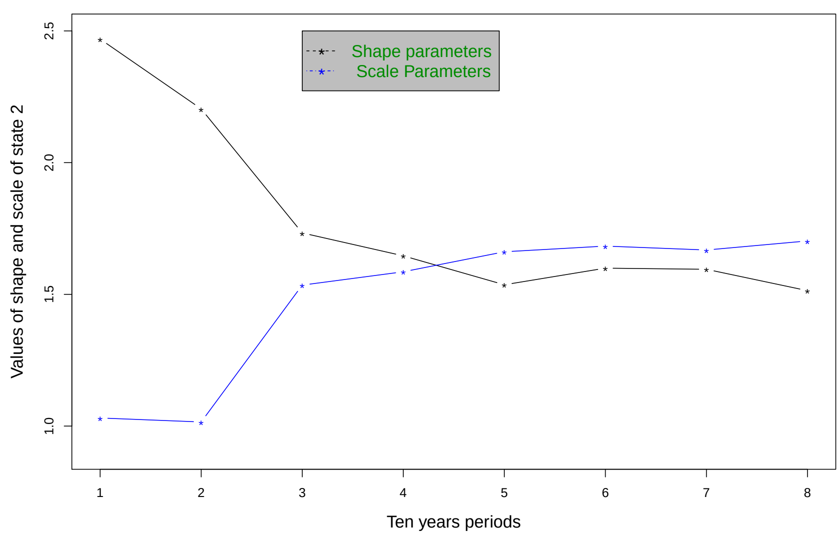

Figure 2: Evolution of shape and scale parameters during ten years periods at Kandi

To determine the beginning and end of the rainy seasons, we estimated the parameters and the best sequence of the dynamic generators over the period from 1921 to 2007. The end and the beginning of rainy seasons estimated by the model with the data of Kandi station are summarized in the figure 3.

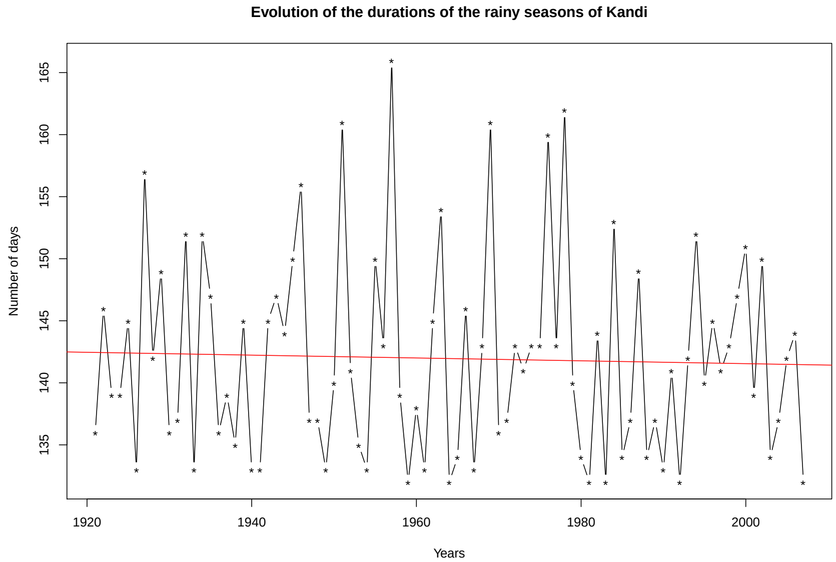

Figure 3: Evolution of the durations of the rainy seasons of Kandi

The trend (red line) of figure 3, shows that the lengths of the rainy seasons are decreasing. We conclude that after 1961 the impact of climate change in Kandi is showing by reduction of quantity and irregularity of precipitation and the reduction of the lengths of rainy seasons.

3.2 Application to data of Parakou

From the results of Fitting the model to each of the 43 sub-data of two-year periods, 17 sub-data of five-year periods, 8 sub-data of ten-year periods, and 4 sub-data of twenty-year periods, we observed :

- at period 19 (1959 – 1960) of two-year periods, 5 (1946 – 1950) of five-year periods (figure 4), the simultaneous reduction of the amplitudes, this means the installation of dry period in rainy seasons, which impact the precipitation;

- the curves of the evolution of the shape parameters μ_2 (for two-year, five-year, ten-year, twenty-years periods) decrease, while the curves of the evolution of scale parameters σ_2 increase, proves that the rainfall decreases at Parakou concerning time and a gap growth between the amount of precipitation from one day to another, more simply the level of bad quality of the rainy seasons' increases concerning time. Moreover, after period 23 (1968 – 1969) of two-year periods, 9 (1967 – 1971) of five-year periods, 4 (1961 – 1970) of ten-year periods (figure 5) and 3 (1981 – 2000) of twenty-year periods, the values of the shape parameters (μ_2) is less than the values of scale parameters (σ_2), this mean that after 1970, we have the succession of bad quality of rainy season in Parakou.

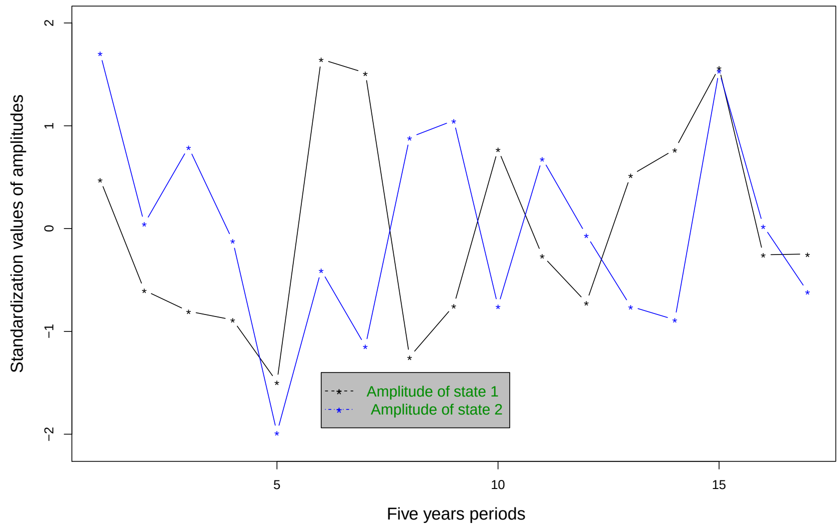

Figure 4: Evolution of amplitudes parameters on five years periods at Parakou

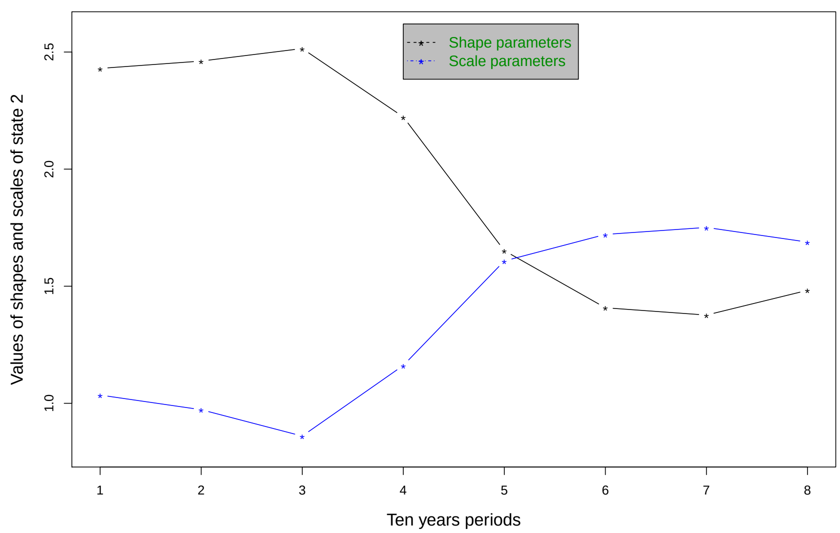

Figure 5: Evolution of shape and scale parameters on ten-year periods at Parakou

From the estimation of the best sequence of precipitation dynamics, the end and the beginning of rainy periods estimated by the model with the data of Parakou station are summarized in the figure 6.

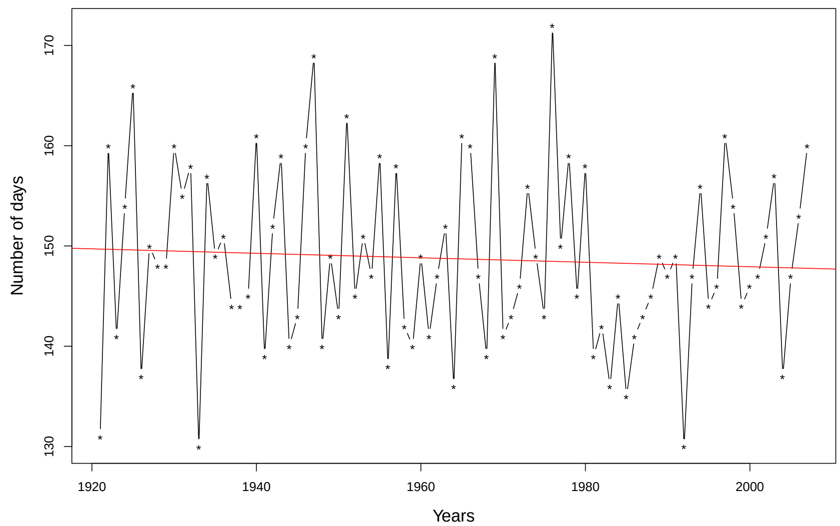

Figure 6: Evolution of the duration of the rainy seasons in Parakou

One observed that the duration of the rainy seasons is relatively identical in time. So, around the years 1970 at Parakou, we have a succession of bad rainy seasons due to the onset of drought in these rainy seasons. Note that the simultaneous reduction of the amplitudes is noticed before 1961, i.e. the installation of the dry sequences started early, and was accentuated in the following years, which also explains the situation after the years 1970. So the fact of climate change at Parakou, the years before 1970 was wetter than years after 1970.

3.3 Application to data of Savè

In the same way as Kandi and Parakou, we observed at Savè that:

- at period 29 (1979 – 1980) of two-year periods, 8 (1961 – 1965) of five-year periods (figure 7),5 (1971 – 1980) of ten-year periods, 3 (1981 – 2000) of twenty-year periods, the simultaneous reduction of the amplitudes, this mean the installation of dry period in rainy seasons, which impact the precipitation;

- the curves of the evolution of the shape parameters μ_2 (for two-year, five-year, ten-year, twenty-years periods) decrease, while the curves of the evolution of scale parameters σ_2 increase, proves that the rainfall decreases at Savè concerning time and a gap growth between the amount of precipitation from one day to another, more simply the level of bad quality of the rainy seasons' increases concerning time. Moreover, after period 15 (1951 – 1952) of two-year periods, 6 (1951 – 1955) of five-year periods, 3 (1951 – 1960) of ten-year periods (figure 8) and 2 (1951 – 1970) of twenty-year periods, the values of the shape parameters (μ_2) is less than the values of scale parameters (σ_2), this mean that after 1961, we have the succession of bad quality of rainy season in Savè.

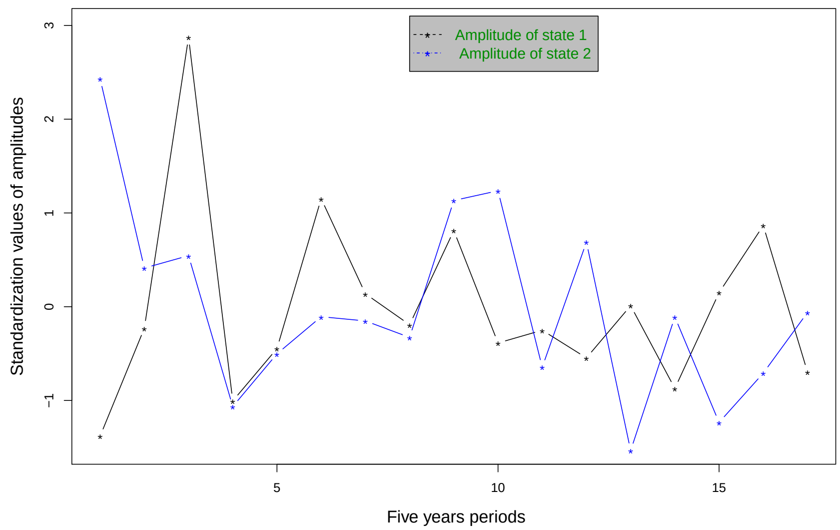

Figure 7: Evolution of amplitudes parameters in five-year periods in Savè

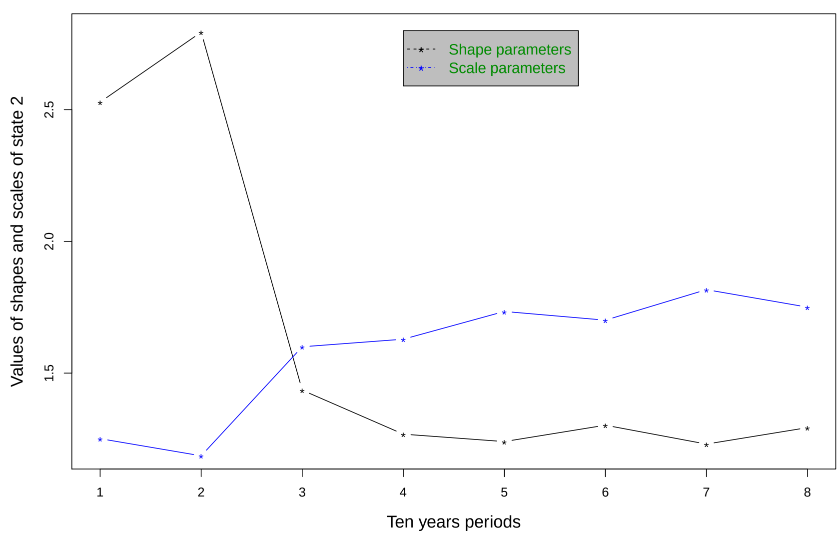

Figure 8: Evolution of shape and scale parameters in ten-year periods in Savè

To determine the start and end of the rainy seasons, we estimated the parameters and the best sequence of generating dynamics over the period from 1921 to 2007 at the Savè station. Figure 12 shows the evolution of the duration of the rainy seasons in Savè.

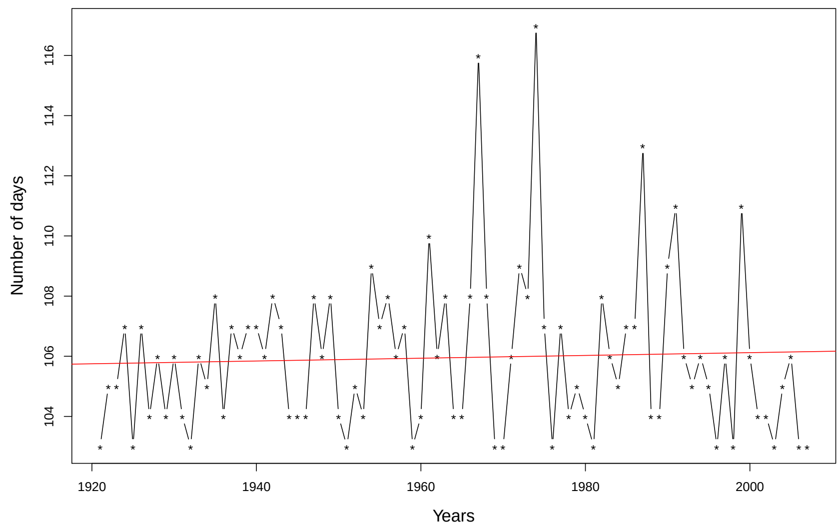

Figure 9: Evolution of the duration of the rainy seasons in Savè

From the figure 12 we notice a slight increase in the duration of the rainy seasons over time. This means that the climatic disturbances observed in terms of the reduction in the amount of precipitation have not affected the duration of the rainy seasons in Savè.

3.4 Application to data of Natitingou

Also at Natitingou, from the estimated results we observed :

- at period 22 (1965 – 1966) of two-year periods, 13 (1986 – 1990) of five-year periods (figure 10), 5 (1971 – 1980) of ten-year periods, 2 (1961 – 1980) of twenty-year periods, the simultaneous reduction of the amplitudes, this mean installation of dry period in rainy seasons, which impact the precipitation;

- the curves of the evolution of the shape parameters μ_2 (for two-year, five-year, ten-year, twenty-year periods) decrease, while the curves of the evolution of scale parameters σ_2 increase, this proves that the rainfall decreases at Natitingou concerning time and a gap growth between the amount of precipitation from one day to another, more simply the level of bad quality of the rainy seasons' increases concerning time. Moreover, after period 28 (1977 – 1978) of two-year periods, 12 (1981 – 1985) of five-year periods, 6 (1981 – 1990) of ten-year periods (figure 11) and 3 (1981 – 2000) of twenty-year periods, the values of the shape parameters (μ_2) is less than the values of scale parameters (σ_2), this mean that after 1981, we have the succession of bad quality of rainy season in Natitingou.

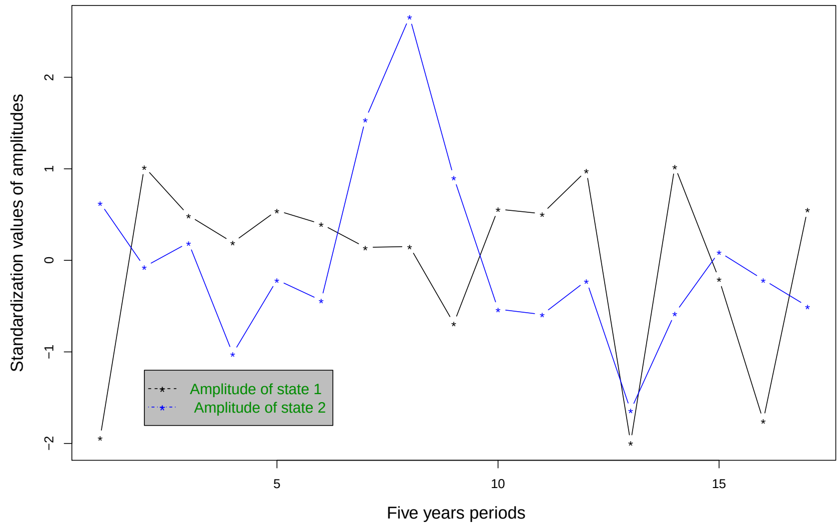

Figure 10: Evolution of amplitudes parameters in five-year periods in Natitingou

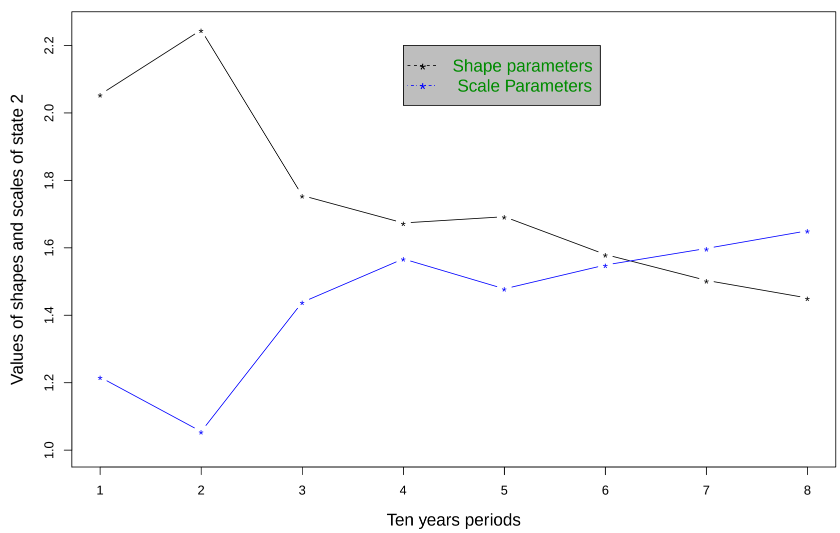

Figure 11: Evolution of shape and scale parameters of ten-year periods at Natitingou

To determine the start and end of the rainy seasons, we estimated the parameters and the best sequence of generating dynamics over the period ranging from 1921 to 2007 at Natitingou station.

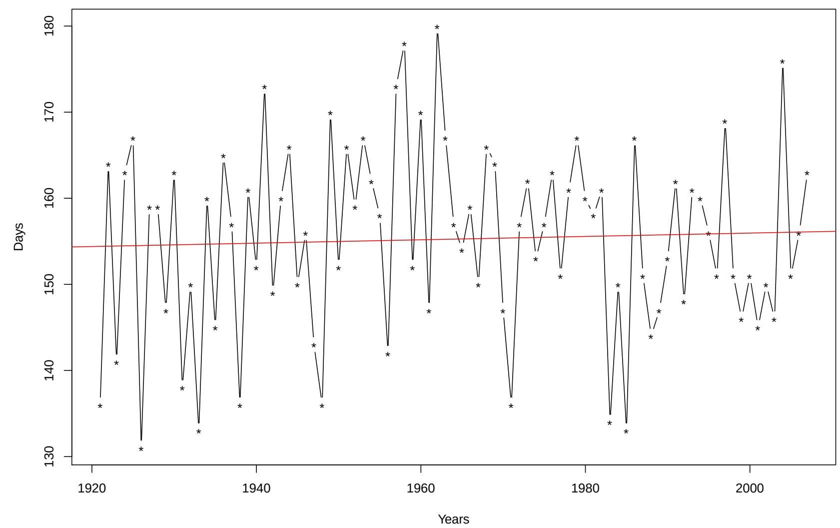

Figure 12: Evolution of the duration of the rainy seasons in Natitingou

The trend (red line) of figure 3, shows that the lengths of the rainy seasons are weakly increasing. We conclude that after 1981 the impact of climate change in Natitingou is showing by reduction of quantity and irregularity of precipitation.

4 Comparative analysis of the stations

All these four stations have a decrease in their quantity of precipitation around 1970 .i.e from the decade 5(1971 – 1980), this impact of climate change has remained more severe over time for some stations than for others. In the rest of the work, we will use the results of the decades (ten-year periods) for the comparative analysis.

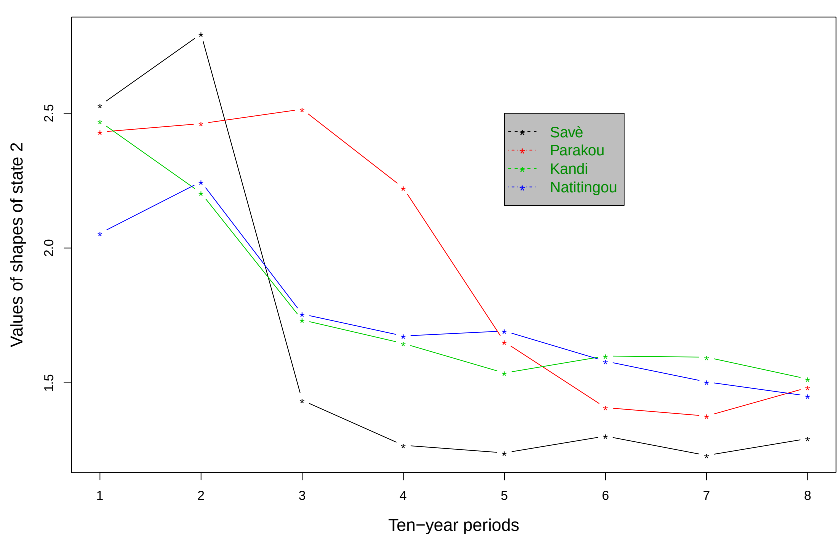

Figure 13: Evolution of shape parameters of the four stations

From the figure 13, we notice that the decrease of the city of Savè is the fastest after follows that of Kandi; the curves of Natitingou and Parakou decrease less quickly. The impact of climate change was quickly noticed at Savè, and its shape ten-year parameter, which was stronger, became over the decades the weakest of the group. As for the Kandi station, after decade 5, the decrease in the shape parameter has become so small that it is higher than those of Parakou and Natitingou, so although Kandi is affected by climate change, it has kept a good rainfall compared to other cities.

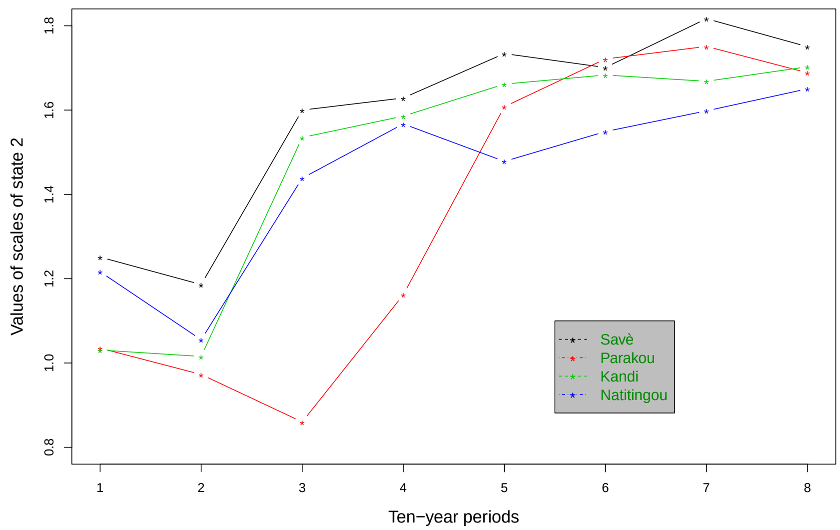

Figure 14: Evolution of scale parameters of the four stations

The figure 14, also shows, that the city of Savè is the most impacted by climate change, and comes after Kandi. Note that after a decade 5, the Kandi curve shows certain stability, which again shows that despite the impact of climate change, the Kandi region has relatively less stable rainy seasons compared to the past than other cities.

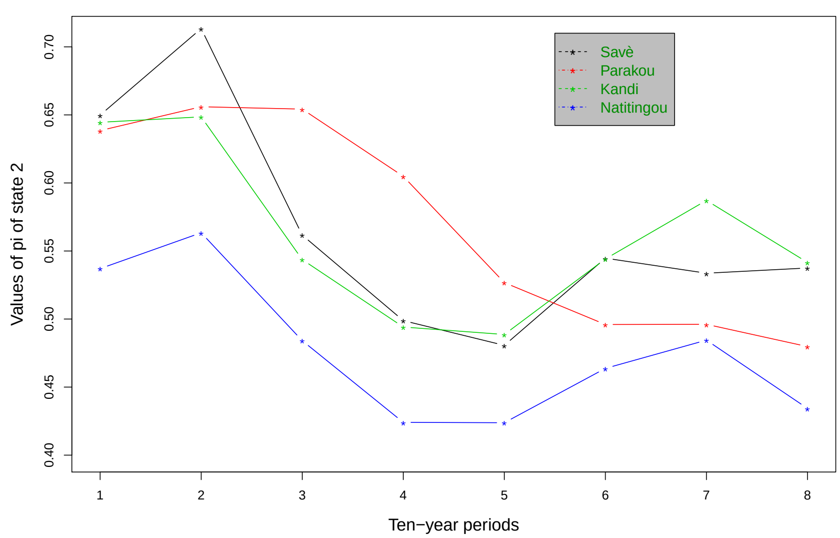

Figure 15: Evolution of parameters π_2

From figure 15, we notice that after a decrease in the values of the "probabilities that it does not rain in state 2", we observe a growth for almost all the curves from period 5 (the periods impacted by climate change). Only the Parakou curve remains decreasing. The towns of Kandi and Savè are the most affected.

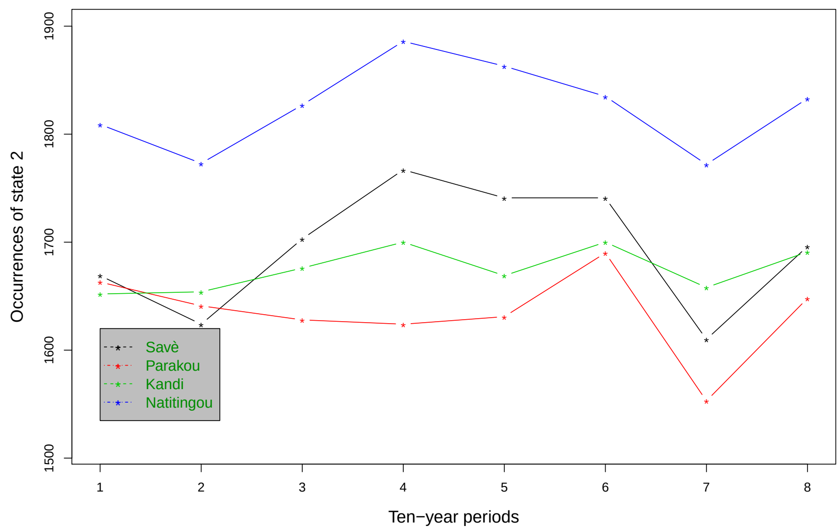

Figure 16: Evolution of occurence of state 2

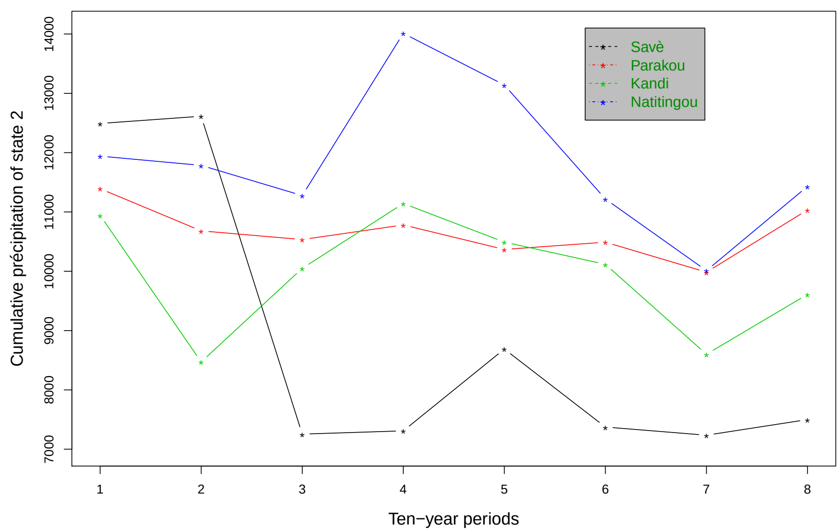

Figure 17: Evolution of cumul of precipitations of state 2

Regarding the occurrences of state 2, Natitingou have the best values, while Parakou and Kandi have the weakest values (figure 16), it does not rain as much in these two cities (table 1). For the cumulative rainfall of state 2, Savè is by finish the lowest, after comes Kandi, Parakou, and Natitingou with the highest values (figure 17).

5 Concluding remarks

From the analysis of the estimate's parameters of the model, we can therefore conclude that:

- all these four cities were affected by climate change after 1970, and the years before 1970 are wetter than those after 1970;
- the towns of Savè and Kandi are affected more quickly than the other two, but despite this, the town of Kandi has kept a better rainfall.

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