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Early schooling to shape the basic elements of probabilistic literacy

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Abstract:
Proportional thinking can be fostered in primary school before fractions are treated in secondary school, so as to foster probabilitic competencies of children at an early age. The study on which the contribution is centered, is based on playful activities and simple written tasks.

Keywords:
Probabilistic thinking, Uncertainty, Proportional thinking, Parts of parts

Introduction:

There is a strong educational interest in fostering children's proportional reasoning, not only because it plays an essential role for the understanding strictly mathematical constructs, but by virtue of the fact that it is a necessary basis for fundamental scientific concepts such as speed, density or electric circuits. All these concepts are similar insofar as they express relationships between "two dimensions" and their appropriate handling requires the knowledge of strategies of proportional reasoning. Even our everyday expertise requires an accurate handling of relations, for instance when comparing prices. Proportional reasoning as a key feature of the formal-operational stage (The fourth developmental stage of Piaget's cognitive theory; for a detailed description see Gudjons 2008) gives an insight into the understanding of comparison and covariation. Furthermore, it serves as an indicator for flexible thinking, which facilitates the involvement of new information and conclusions rather than making use of existing information (Koerber, 2003, p.76; Hafner, 2013, p.14). They are used in all sorts of daily tasks and, above all, they are at the basis of inference and prediction. Proportional thinking is the basis for dealing with frequencies and probabilities and risks (Bryant & Nunes, 2012; in Till, 2014, p. 58). There is a strong interest in proportional reasoning, not only because it plays an essential role for the understanding of algebraic concepts, but by virtue of the fact that it is a necessary basis for fundamental scientific concepts such as speed, density or electric circuits. All these concepts are similar insofar as they express a relationship between two dimensions and their appropriate handling requires the knowledge of strategies of proportional reasoning. We briefly mention the additive misconception, well-known to Piaget and different strategies to learn to work with proportions.

A proportion refers to the part of the whole that has certain characteristics of interest. It can be written in form of a fraction or ratio, that is, a/b or a : b and expressed in words as "a out of b", like in "three out of four children are eating apples". In mathematics, two variables are proportional if a change in one is always accompanied by a change in the other, and if the changes are always related by use of a constant multiplier. Or in other words, a proportion is defined as equivalence between two relations (2 out of 4 = 4 out of 8 = 50%) (Koerber, 2003). Proportions are widely applied in probabilistic problems, especially when frequencies, or, more precisely, expected frequencies are used, like in the sentence: "Three of each four children, usually like bananas".

Secondary school pupils are usually taught to their build the ability to think and compare two quantities by using multiplicative thinking. Accordingly, this knowledge enhances the cognitive power of pupils in storing and processing relations. Thus, it includes both quantitative and qualitative methods of thought.

As an illustration, in rolling a dice, the random probability of getting any specific number is 1/6 or 1 : 6. Similarly, if there are twelve balls in an urn six blue and six black ones, the probability of each coloured ball is 1/12. By drawing one ball from the urn at a time, the total number of balls will decrease and the probability to draw one specific colour will increase. For instance, after drawing 2 balls from the urn, the probability of drawing the required colour from the remaining ones is 1/10. The total probability is computed by multiplying the set of proportions, making use of proportional reasoning techniques that improve pupil's ability to memorise things and relations.

In case a of fair die with six faces, the probability of getting any number from 1 to 6 when rolling the die, does not change, but remains the same and can be expressed as "1 out of 6".

In probabilistic problems, different types of scenarios may involve involved proportional thinking, namely when dealing with conditional probabilities.

The additive misconception

The concept of proportions is an inevitable basis for statistics and probability calculus (Bryant & Nunes, 2012; Martignon & Krauss, 2007). Nevertheless, proportional reasoning is rather difficult in general – not only for children, but also for adults. As Bryant and Nunes (2012) stated, tasks in which children have to compare two or more different probabilities most clearly illustrate this difficulty. When it comes to quantifying proportional problems, younger children tend to mistakenly apply additive strategies, which means that they consider the relation within and between the proportions erroneously as a difference between terms instead of a multiplication. The most difficult problem of the development of a proportional understanding seems to be the conversion of mathematical strategies from additive to multiplicative methods (Koeber, 2003).

One example which clearly illustrates the additive misconception is the urn task of PISA 2003.

> "Box A contains one white and two black marbles. Box B contains two white and five black marbles. You have to draw a marble from one of the boxes with your eyes covered. From which box should you draw if you want a white marble?" (PISA-Konsortium, 2004, p.60).

The solution is not to be found in the absolute numbers of the two colours, but in the proportion of white balls in each box. PISA 2003 showed that 73 % of 15- year old pupils mistakenly applied additive strategies. The equivalent relation of 1 out of 3 is 2 out of 6. Multiplying both values (1 and 3) with the factor 2 leads to the equivalent relation 2 out of 6. People, then, fallaciously assume that the addition of 1 to both entities of the relation (1:3) will lead to an equal proportion (1 (+1) *out of* 3 (+1) = 2 *out of* 4). The additive misconception, however, is not only applied by younger children, but also by adolescents and adults (Koerber, 2003).

Research by Piaget and Inhelder (1975), Falk et al. (1980), Fischbein and Gazit (1984), Falk and Wilkening (1998), Falk et al. (2012) and Dension and Xu (2013), has established that pupils get better at making proportional calculations while working with probabilities as they grow older.

Experiments on proportions for fostering proportional and probabilistic thinking

Inspired by the PISA task described above a master student at the PH Ludwigsburg played with 42 fourth-graders, performing repeated urn games, based on extraction of coloured balls from urns with different proportions. Here the emphasis was not on proportions but on the "entropy" of the distributions of colours involved. Her purpose was, in fact, to foster intuitions about the entropy of distributions (Özel, Nelson, Bertram, Martignon, 2021). After a brief intervention children were tested with tasks like the following (we report on two tasks only, which are of interest for our purpose):

Task 1 :

"Below you see two jars. Each jar contains a specific number of white and black marbles. The jar on the left (jar 1) contains one white and two black marbles. The jar on the right (jar 2) contains two white and five black marbles.

Somebody mixes up the jars and draws a marble without peeking.

If you win a prize by drawing a white marble, which jar would you choose? "

jar 1 — 2 black marbles and 1 white marble

jar 2 — 5 black marbles and 2 white marbles

Which jar have you chosen? Explain your decision!

Task 3 :

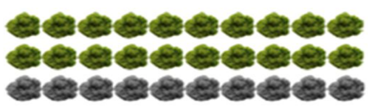
„Joachim's aunt and uncle have a garden with lettuces growing. Unfortunately, snails have bitten some of those lettuces (bitten snakes are grey in Figure 8 B below) .

- In your aunt's garden 10 out of 30 lettuces were eaten up by snails.
- In your uncle's garden 20 out of 100 lettuces were eaten up by snails.

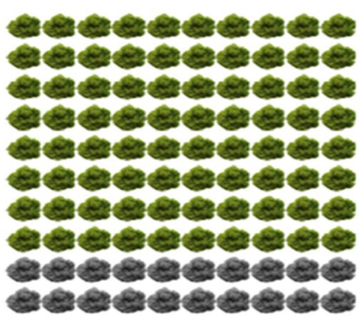
In which garden relatively more lettuces were eaten up?

- o In your uncle's garden
- o In your aunt's garden

Explain your decision!"



„Lettuces in the aunt's garden."



„Lettuces in the uncle's garden."

Figure 1. Two tasks for assessing proportional thinking competencies

The results were interesting from different perspectives, and are summarized in Figure 2.

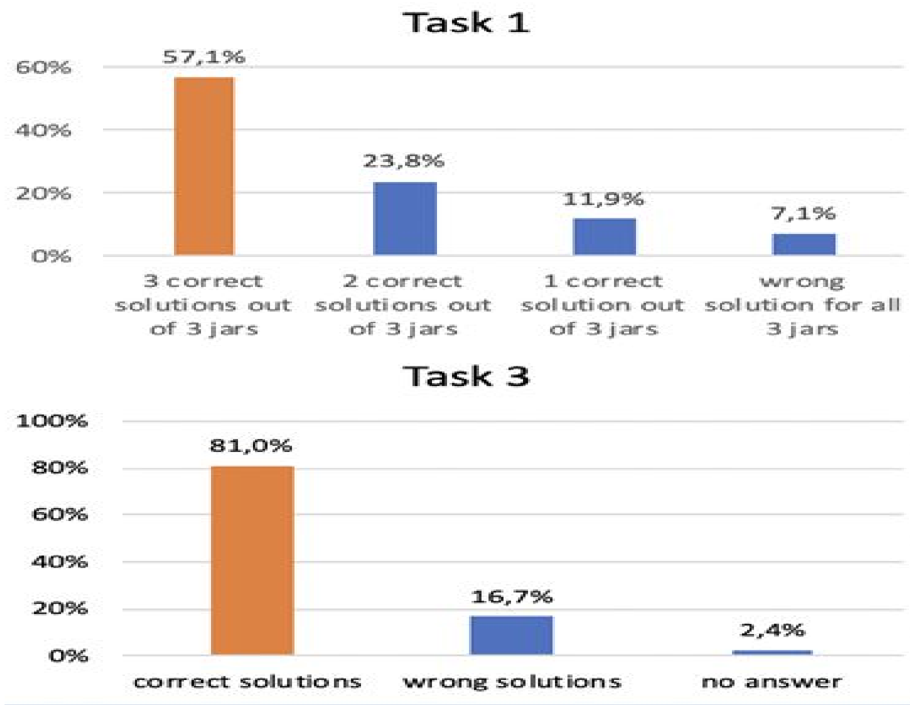


Figure 2. Graphs exhibiting the proportion of correct solutions

Özel also analysed and categorized children's answers, as illustrated in Figure 3 and 4:

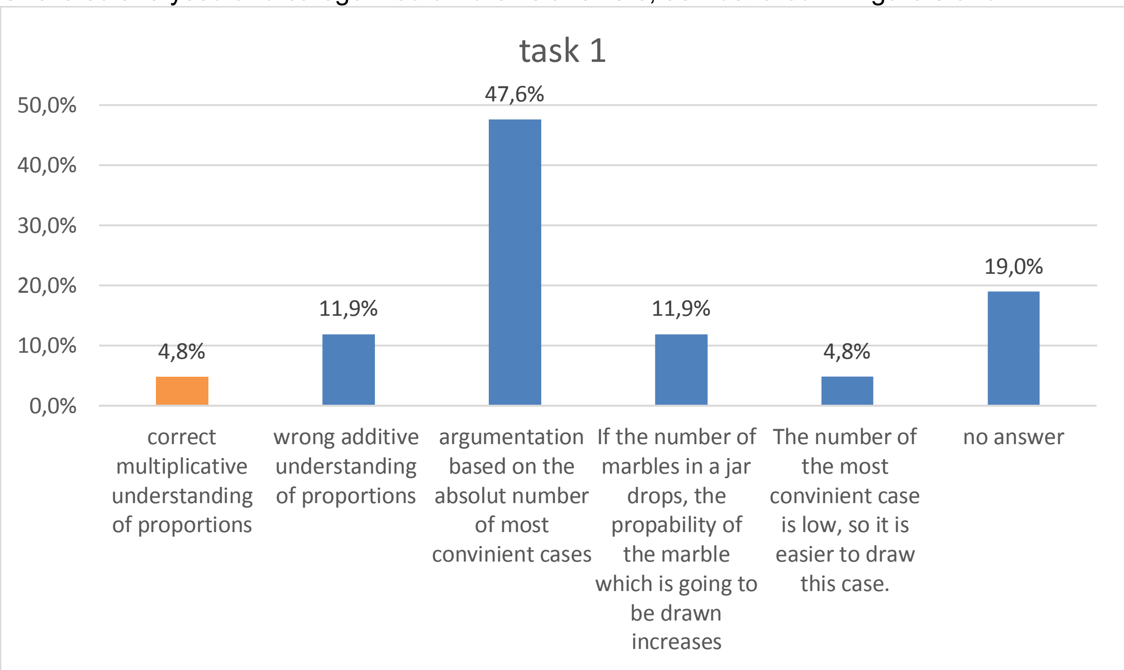


Figure 3. Categorization of children's answers for Task 1

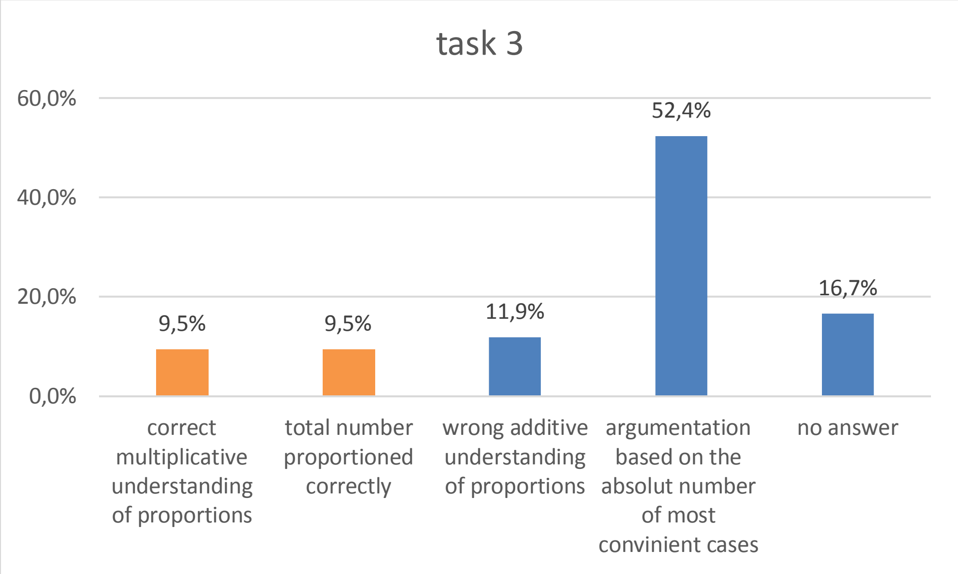


Figure 4. Categorization of children's answers for Task 3

The important observation extracted from this empirical work is that it makes great sense to develop a sense for proportions previous to the treatment of fractions and to couple it with a basic understanding of elementary probabilities. In fact it is the development of a sense for "parts of parts" and the estimate of the size of these relations between the part and the whole, that establishes the basis for conditional probabilities, expressed in terms of "natural frequencies" (Martignon, 2014).

References:

Bryant, P. & Nunes, T. (2012). *Children's understanding of probability. A literature review*. www.nuffieldfoundation.org/sites/default/files/files/Nuffield_CuP_FULL_REPORTv_FINAL.pdf

Denison, S. & Xu, F. (2013). The origins of probabilistic inference in human infants. *Cognition, 130*(3), 335-347.

Falk, R. & Wilkening, F. (1998). Children's construction of fair chances: Adjusting probabilities. *Developmental Psychology, 34*(6), 1340-1357.

Falk, R., Falk, R., & Levin, I. (1980). A potential for learning probability in young children. *Educational Studies in Mathematics, 11*, 181-204.

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? An exploratory research. *Educational Studies in Mathematics, 15*(1), 1-24.

Koerber, S. (2003). *Visualisierung als Werkzeug im Mathematik-Unterricht. Der Einfluss externer Repräsentationsformen auf proportionales Denken im Grundschulalter*. Hamburg: Verlag Dr. Kovac.

Martignon, L. (2014). Fostering children's probabilistic reasoning and first elements of risk evaluation. In: Chernoff, E.J & Sriraman, B. (Eds.), *Probabilistic thinking: Presenting plural perspectives* (pp. 149-160). New York: Springer.

Martignon, L. & Krauss, S. (2009). Hands-on activities for fourth graders: A tool box for decision-making and reckoning with risk. *International Electronic Journal of Mathematics Education (IEJME), 4*(3), M. Borovcnik & R. Kapadia (Eds.), Special issue on "Research and Developments in Probability Education", 227–258.

Özel, E, Nelson, J, Bertram, L., Martignon, L. (to appear) Frontiers: Psychology and Mathematics Education.

Piaget, J. & Inhelder, B. (1975). The Origin of the idea of chance in children. London: Routledge and Kegan Paul.

Pisa Konsortium. (2004). *Pisa 2003: Der Bildungsstand der Jugendlichen in Deutschland.* Ergebnisse des zweiten internationalen Vergleichs. Münster: Waxmann.

Till, Ch. (2015). Entwicklung der Vorstellung von Grundschülerinnen und Grundschülern zu Risiko und Entscheidungen unter Unsicherheit. Dissertatio, PH Ludwigsburg.