Development of optimal weights for cyclical composite indicator systems based on multivariate spectral analysis of time series.

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Summary: In this work we propose a method to develop sets of optimal weights for the development of cyclical indicators that are also optimal. For this we resort to the canonical analysis of multivariate time series in the frequency domain, following Brillinger (1981). To carry out useful comparisons, we make use of the sets of coincident and leading indicators, in use by the Cyclical Indicators System of INEGI, for the period January 2004 to March 2020. The first application of the suggested procedure to the data set it does not result in optimal behavior. We will show how our procedure allows evaluating the candidate indicators to enter the analysis, which will result in refined sets. From the result of this selection process, a new application of the methodology is made. Based on criteria such as the cross-correlation to different lags, or by the ability to make forecasts of the coincident indicator from the advance, we evaluate the profit to which our proposal leads. The results obtained in this way exceed in more than one sense the behavior of indicators obtained using traditional methods.

Keywords: composite cyclical indicators, multivariate time series, spectral analysis, canonical correlation, forecasts.

Introduction

The methodologies currently in use for the development of systems of cyclical indicators, coincident and leading, have their origin in the pioneering work of Messrs. Mitchell[1] and Burns[2], of the National Bureau of Economic Research of the United States, during the first third of the 20th century. In addition to their disciple Geoffrey Moore[3], various national and international organizations have followed their footsteps and made contributions that have the purpose of improving methodologies and results. From 1995, The Conference Board (see TCB (2001)) has continued to publish periodic results (see TCB (2020)), both for the United States and for other countries. However, the fundamental aspects of the original proposal remain in all of them.

For instance, the recognition of the inability of a single indicator to provide an adequate summary of the recent cyclical behaviour of a country's economy remains. That is why, in all cases, they seek to identify sets of coincident indicators, whose aggregation is expected to better approximate such behaviour. In general, this identification is carried out by dating changes of direction in the evolution of each indicator and comparing said dating with that of a reference indicator that collects the aggregate behaviour of production in the productive sectors, desirably GDP or, preferably, some monthly indicator of industrial activity. When the direction changes of the ridges or in the valleys coincide with great frequency, another coincident indicator is said to have been identified. In a similar way, a second set of indicators, called leading, is formed whose peaks and valleys occur in time, generally before those of the reference indicator.

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[1] https://www.nber.org/people/wesley_mitchell?page=2&perPage=50
[2] https://www.nber.org/people/arthur_burns?page=2&perPage=50.
[3] https://www.nber.org/people/geoffrey_moore?page=1&perPage=50.

The information contained in each one of the sets developed as described, or that of their growth, is summarized by means of the weighted sum of the contemporary values of the different indicators to obtain composite indicators. Over time, weighting schemes have been simplified. Originally, the dating of ridges and valleys, as well as the determination of the coefficient sets, took place through meetings of experts during which agreements were sought. Today, general practice indicates that the mentioned weights be based on the variability of each of the indicators to avoid giving greater weight to those that show greater heterogeneity during the observation period.

A recent addition to the methodology consists of the use of versions of the indicators whose secular trend has been reduced or outright eliminated. For this purpose, proposals such as the Hodrick and Prescott filter (Hodrick et al. (1997)) have been used or, as in the case of the OECD (see OECD (2012)), considerations of the spectral analysis of time series; in this case, one proceeds by reducing the power at smaller frequencies.

The experience in the use of methodologies such as the one we have summarized in previous paragraphs is rather mixed. Since one of the main aims of such methodologies is to anticipate undesirable drops in economic performance to act with opportunity to mitigate their negative effects on people and companies, it would seem sensible to choose the one among all the available options leading to smaller forecast errors. In the recent past, this would have been of great importance in the face of a crisis like the one that unfolded towards the end of 2008 and whose residual effects are still perceptible today. If we judge by the magnitude of what happened then, it would seem that the obtained forecasts did not account for it, or that the economic policy decision makers ignored them. In the Mexican case, INEGI determined to change the methodology that it had been following until before 2009 (see INEGI (2015)) based on that experience (see Yabuta (2010)).

It is possible to find in the literature attempts that seek the incorporation of more modern tools of the econometric analysis of time series in the elaboration of coincident and leading indicators. For example, Stock et al. (1989) seek to answer three basic questions. From a conceptual perspective they wonder if it is possible to develop a formal probabilistic model that gives rise to the indices and allows their evaluation. They also ask about the best set of variables to be included in the leading indicator. Finally, they seek to investigate the best way to combine these variables to produce useful and reliable indexes. Its methodology consists of two stages and starts from the identification of a set of coincident variables from which a single dynamic factor is obtained, common to all of them, which will be used as a coincident indicator. Then, and under the consideration of a broad set of identifiability conditions, they model the stochastic vector formed by the dynamic factor and the leading variables as a Vector Auto-Regression, paying particular attention to the equation for the dynamic factor. They exhibit examples of the application of their proposal making use of the information available for the construction of the coinciding and leading indicators prepared by the NBER.

We will address what we consider to be the most important limitations of the currently most widely used methodologies throughout the remainder of this section. It is striking that methodologies that were born almost a century ago have not incorporated the theoretical and methodological developments of the statistical analysis of multivariate time series that have been published since then, nor the appreciable strengthening of currently available information technologies. It may be that for regulatory aims the dating of ridges and valleys is required to enable policy makers to act to mitigate the consequences of a fall, or to stop doing so when it is no longer required. However, reasonable practices of time series analysis would suggest also taking advantage of the information that is presented between them, through the patterns of auto correlation and cross correlation. One consequence of proceeding in this way would be that the dating of the turning points, perhaps the only possibility of relating two indicators considering the calculation methodologies available at the beginning of the 20th century, would lose relevance in today's context.

Nor can the absence of an optimality criterion be ignored since they allow establishing better values for the coefficients of the indicators. Today, those of one set seem to be determined without considering the information contained in the other. Obtaining an indirect forecast (to distinguish it from self-projections) with reasonable precision does not seem to be relevant for this aim either.

Surely when the aim is to contrast the values of an indicator at one time with the values of the same at another time, it is necessary to restrict the aggregation of values only to contemporary ones; for example, that is the custom for Laspeyres or Paasche indices. The question arises whether when the purpose is different, as has been established, it is reasonable or desirable to maintain the restriction to contemporary values. The popularization of the ARIMA models (Box and Jenkins (1970)), for the univariate case, or the Vector Auto Regressions (VAR), for the multivariate, is because they improve the precision of the forecasts through the consideration of lags other than zero.

The proposal summarized in Bustos (1993) already offered answers to several of the previous concerns and limitations. His main source of inspiration is Brillinger (1980, Ch. 10), specifically his treatment of Canonical Time Series Analysis, taking advantage of the frequency domain perspective. The basic reasoning behind the proposal was that, when studying the business cycle or the economic cycle, it is natural to approximate the time series that represent it by means of weighted sums of periodic functions with stochastic coefficients, in a range of frequencies; that is, its Fourier transforms. Some recent applications of this approach to econometric analysis can be found in Pollock (2014a, 2014b, 2018).

The advantages of proceeding in this way are diverse. First, this approach does not lead to any loss of information; it simply corresponds to a complementary analysis of the same information from a different perspective. Furthermore, in the stationary case, the second moments are simplified. In particular, the foregoing gives the possibility of transferring some multivariate techniques of the statistical analysis of random vectors, applying them frequency by frequency. Similarly, the secular trend common to all the series can be removed by reducing the power at the low frequencies of the spectrum of the first principal component, obtained from all the series under consideration. Stationary versions of the original series are derived from the set of modified principal components.

Another consequence of the above is that contrary to custom, explicit criteria are introduced to evaluate the quality of the result; for example, canonical analysis seeks to determine vectors of coefficients that maximize the square of the norm of coherence at each frequency. It is expected that, upon returning to the time domain, the results will also exhibit optimal cross-correlations for one or more lags. It is therefore expected that the forecasts provided by the leading indicator for the coincident indicator will turn out to be optimal in some sense.

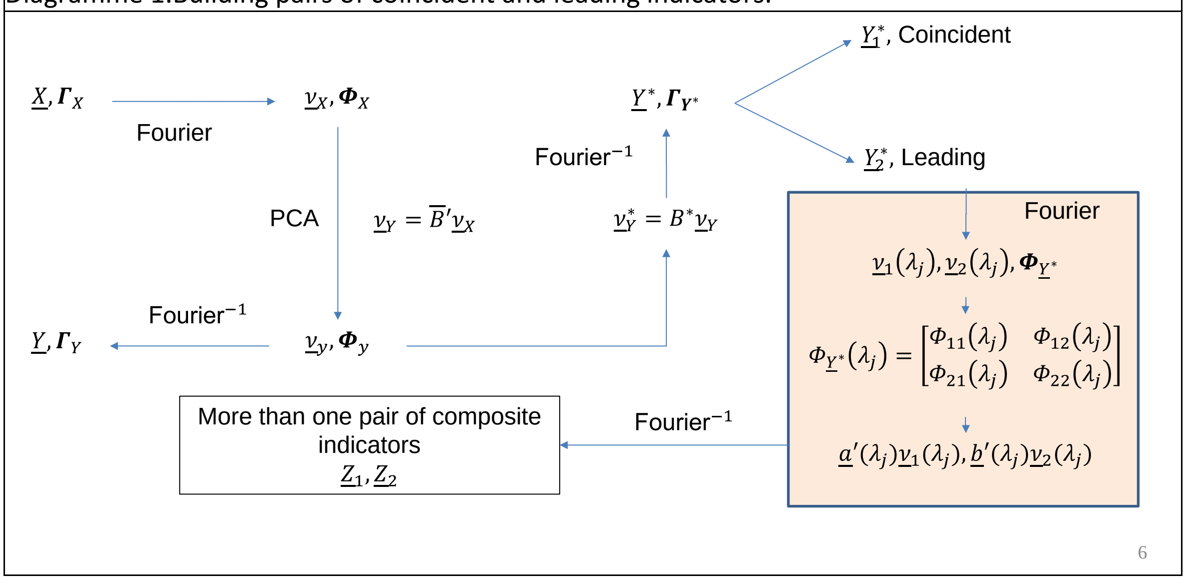
Another limitation that that proposal sought to study refers to the assumption, not always corroborated, that a single composite indicator for each set summarizes all the information related to the cycle, contained in the original data. Thus, by giving rise to more than one set of pairs, the suggested approach allows us to appreciate the optimal reduction of the dimensions.

The rest of this work proceeds as follows. In the following section, a brief outline of the proposed methodology is prepared, as well as some relevant results. We will then proceed to apply our proposal to the information used in the construction of the SIC-INEGI. In this way, we will be able to carry out a comparison between both sets of results, introducing for this purpose the criteria that we consider appropriate. Next, and taking advantage of the criteria introduced, we will propose a strategy to eliminate undesirable indicators from the set of coincident ones. Something similar will happen with the leading. Finally, with the result of the previous selection process, a new application of our methodology will be carried out and its result will be compared with that of the procedure currently in use by INEGI. We will conclude with a brief summary of our results and with the discussion of some approaches for future research.

Canonical analysis of multivariate time series

A sketch of the complete proposal for the calculation of a system of cyclical indicators can be found in diagram 1. In it, two parts can be identified to carry out the analysis. First, starting from the original information of the set of n series $\{\underline{X}_t\}_{t=1}^{T}$, as well as their second moments $\{\boldsymbol{\Gamma}_X(k)\}_{k=0}^{T-1}$, analysis by principal components is performed in the frequency domain, if the circumstances so require. Then, starting from information derived from the previous analysis or from the original information itself, the canonical analysis itself takes place, also in the frequency domain.
In all cases their equivalent versions are found in the frequency domain, $\underline{\nu}_X(\lambda), \boldsymbol{\Phi}_X(\lambda)$; in essence, the Fourier transforms, finite and discrete. For each of the frequency values $\lambda_j = 2\pi j/T, j = 0, \dots, T-1$, starting from the diagonalization of the Hermitian spectral matrix $\boldsymbol{\Phi}_X(\lambda_j)$, the principal component analysis is carried out. In other words, by means of the matrix B whose columns are the characteristic vectors of the spectral matrix, a new set of Fourier coefficients is obtained whose spectral matrix turns out to be diagonal. Through the inverse Fourier transform we have a new set of series $\{\underline{Y}_t\}_{t=1}^{T}$, as well as its covariance structure $\{\boldsymbol{\Gamma}_Y(k)\}_{k=0}^{T-1}$, which is approximately formed by diagonal matrices. In this way, it is achieved that the new set of series has null cross-correlations, similar to what happens in the analysis of principal components of independent random vectors.
The principal component analysis also allows reducing the influence of some frequencies in new versions of the original series. When these frequencies coincide with the smaller ones, the secular trends become less important. This treatment is reminiscent of the one described in OECD (2012, Annex A), with the difference that in its case the elimination of low frequencies takes place series by series.

Diagramme 1.Building pairs of coincident and leading indicators.



The canonical analysis itself proceeds from the original set of series or the one in which the trends have been attenuated, as indicated above. In this case, in order to make efficient use of all the

available information, the determination of the sets of coincident or leading series takes place according to the behaviour of the cross-correlation structures of each and every one of the candidates with a reference indicator. In short, what this means is that neither the pivot points on ridges and valleys, nor their dating, are a relevant part of our approach.
Once the sizes of each set, denoted by p_1 and p_2, have been determined, the spectral matrix of the vector whose first p_1 components are formed by the coincident series, while the remaining p_2 are occupied by the leading series, is estimated. The estimation is carried out as indicated in (1).

$$\Phi_X(\lambda) = \sum_{j=-P}^{P} \frac{P-|j|}{P} \hat{\Gamma}_j \, e^{i\lambda j} \tag{1}$$

Matrix $\Phi_X(\lambda)$ is partitioned into blocks in a conformable manner as indicated in expression (2).

$$\Phi_X(\lambda) = \begin{bmatrix} \Phi_{11}(\lambda) & \Phi_{12}(\lambda) \\ \Phi_{21}(\lambda) & \Phi_{22}(\lambda) \end{bmatrix} \tag{2}$$

Based on it, and for each one of the frequencies λ_j, the optimization problems indicated in (3) are raised, from whose solutions the vectors of coefficients $\underline{a}(\lambda_j), \underline{b}(\lambda_j)$ that lead to the canonical spectral pairs (see Brillinger (1981, Theorem 10.3.2)).

$$\underset{\underline{a}(\lambda),\, \underline{b}(\lambda)}{\text{Max}} \left| Corr\{\underline{a}'(\lambda)\underline{v}_1(\lambda), \underline{b}'(\lambda)\underline{v}_2(\lambda)\}\right|^2 = \underset{\underline{a}(\lambda),\, \underline{b}(\lambda)}{\text{Max}} \left| \frac{\{\underline{a}'(\lambda)\Phi_{12}\underline{\bar{b}}(\lambda)\}}{\sqrt{(\underline{a}'(\lambda)\Phi_{11}\underline{\bar{a}}(\lambda))(\underline{b}'(\lambda)\Phi_{22}\underline{\bar{b}}(\lambda))}} \right|^2 \tag{3}$$

It follows that $\underline{a}(\lambda)$ and $\underline{b}(\lambda)$ maximize the squared norm of the coherence within the canonical pair. We observe that it is possible to obtain new pairs of vectors by imposing conditions of null correlation between canonical pairs to solve optimization problems like (3). This can be carried out up to a number of times equal to min(p_1,p_2). Furthermore, under the condition that the vectors of coefficients obtained this way have a norm equal to 1, these vectors are in turn non-zero solutions to the systems of equations presented in (4). In other words, the vectors $\underline{a}(\lambda), \underline{b}(\lambda)$ are characteristic vectors of the matrices that appear in (4). It can be proven that the corresponding characteristic values $\rho(\lambda)$ coincide with the optimal value of the maximization criterion in (3).

$$\left.\begin{matrix} \Phi_{11}^{-1}(\lambda)\Phi_{12}(\lambda)\Phi_{22}^{-1}(\lambda)\Phi_{21}(\lambda)\underline{a}_j(\lambda) = \rho_j(\lambda)\underline{a}_j(\lambda) \\ \Phi_{22}^{-1}(\lambda)\Phi_{21}(\lambda)\Phi_{11}^{-1}(\lambda)\Phi_{12}(\lambda)\underline{b}_j(\lambda) = \rho_j(\lambda)\underline{b}_j(\lambda) \end{matrix}\right\},\ j = 1, \dots, \min\{p_1, p_2\} \tag{4}$$

From each of these solutions, new sets of Fourier coefficients are defined. Again, the inverse Fourier transform of these results will give rise to new series of indicators whose cross-correlation, it is expected, will be maximum for some lag. A new vector of series $\{\underline{Z}_t\}_{t=1}^{T}$ is formed whose first p_1 components the coincident indicators are concentrated, and in the subsequent p_2, the leading ones. By construction, the calculation of the resulting indicators can consider, in addition to contemporary values, those of lagged values from the original series. In this way, the implicit restrictions on the calculation of indices, which assume that the values of the coefficients for non-null lags are equal to zero, no longer apply. Therefore, it is conceivable that the optimality criterion will show better levels.

Example of application of the methodology and comparison of its results with those of INEGI's System of Cyclical Indicators (SIC).

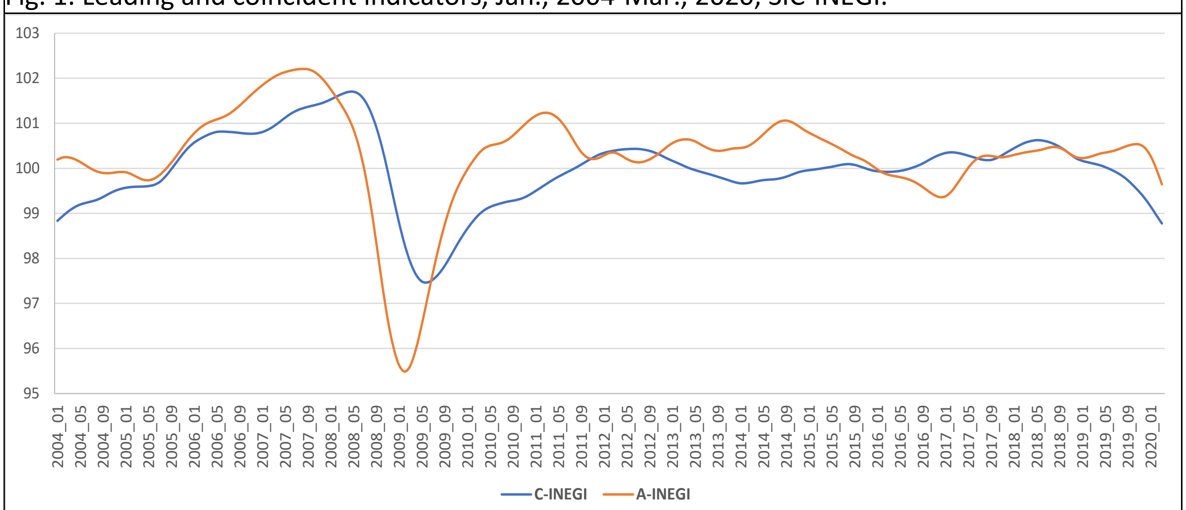
As a practical example of the methodology outlined in the previous section, we will use the series of indicators that INEGI makes available on its Internet page[4]. According to this publication, the original indicators had a trend removed and were smoothed through a double application of the Hodrick and Prescott filter. The period of observation covers 195 observations, from January 2004 to March 2020. Since a detrended version of the indicators is used, comparisons focus only on the determination of weights for the composite indicators. Table 1 lists, along with their mnemonics, the series that are part of the coincident and of the leading sets, and were selected, according to what is indicated in INEGI (2015), dating their peaks and troughs, and comparing them with those of the monthly Global Indicator of Economic Activity (IGAE).

Table 1. Component series, and mnemonics, for SIC-INEGI

Coincident Series	**Leading Series**
1. *Global Indicator of Economic Activity,* (IGAE).	1. *Manufacturing Employment Trend,* (EMPMANUF).
2. *Industrial Activity Indicator,* (ACTIND).	2. *Indicator of Business Confidence,* (CONFEMP).
3. *Supply of Goods and Services Retail Income Index,* (INGXBYS)	3. *Price and Quotation Index of the Mexican Stock Exchange in real terms,* (IPC-BMV).
4. *Number of Permanently Insured, Mexican Social Security Institute,* (IMSS).	4. *Bilateral Real Exchange Rate Mexico - USA,* (TICAM).
5. *Uban Unemployment Rate,* (TDOU)	5. *Equilibrium Interbank Interest Rate,* (TIIE).
6. *Total Imports,* (IMPORT).	6. *Standard & Poor´s 500 Index,* (S&P500).

The results published by INEGI are produced through its methodological proposal and are briefly discussed below. Figure 1 shows the result of adding, according to the methodology used by INEGI, the series that make up each of the previous sets, and their comparison. As can be seen, the leading composite indicator shows a desirable behaviour in terms of peaks and valleys towards the beginning of the period; however, its performance deteriorates near the end. It seems clear for instance that, while the coincident indicator has been declining for almost two years, the leading indicator shows a slight increase during the beginning of that period, and only for the most recent months shows a change in direction.

Fig. 1. Leading and coincident indicators, Jan., 2004-Mar., 2020, SIC-INEGI.

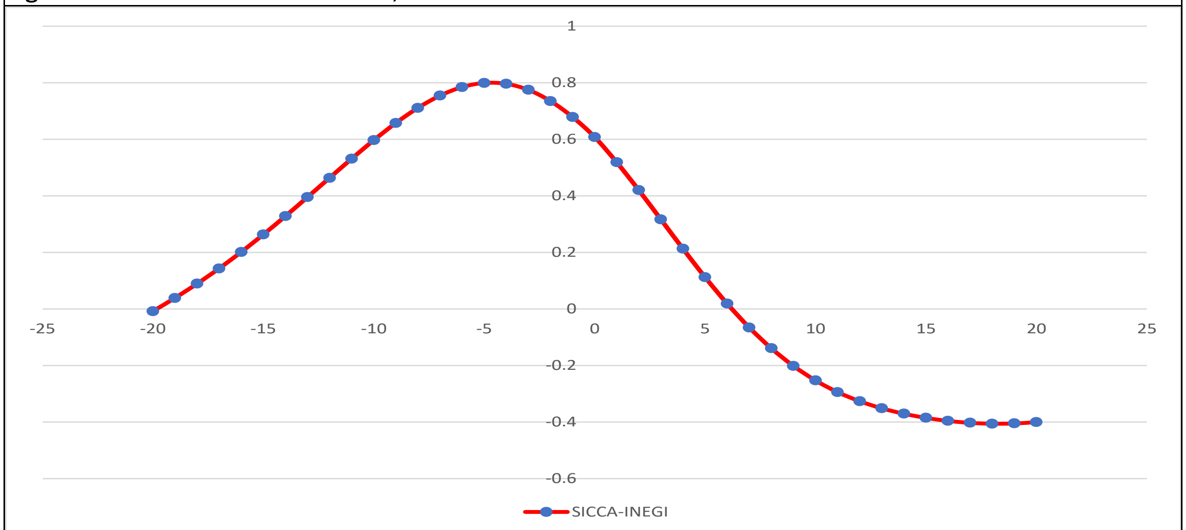


Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

[4] Available at: https://www.inegi.org.mx/app/indicadores/?tm=0&t=10000215#D10000215

As shown in Figure 2, the cross-correlation structure between both indicators shows that the lead between one and the other is, on average, around 5 months throughout the entire period. The value of the cross-correlation at this lag is equal to 0.7921, being significantly different from zero whatever the test criteria. This indicator will be useful for making comparisons between both procedures, and its value represents the benchmark against which we will measure the performance of the alternatives.

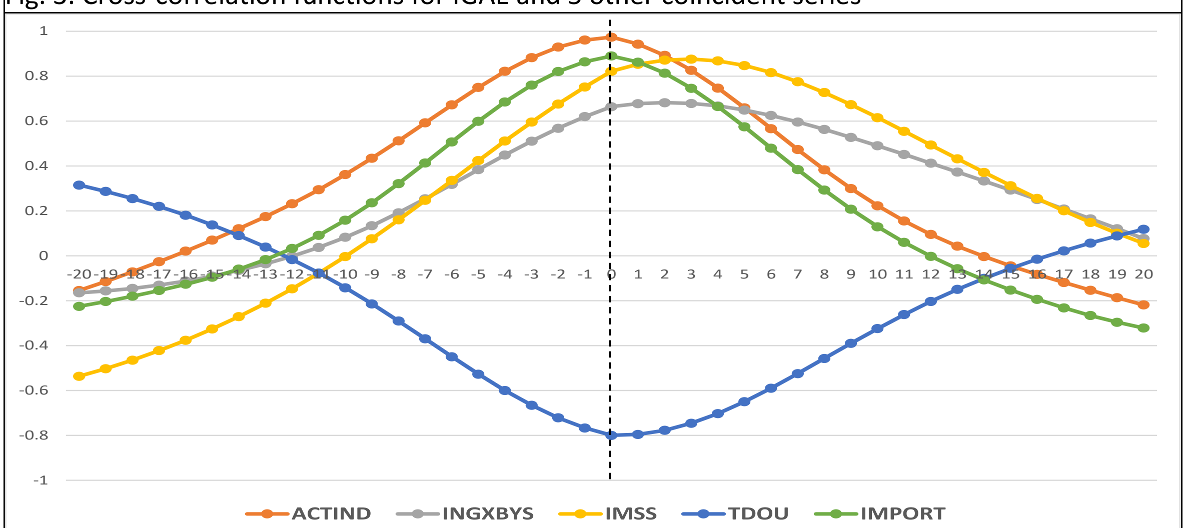
Fig. 2. Cross-correlation function, SIC-INEGI



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

As already mentioned, and to make efficient use of available information, consideration of the cross-correlation patterns between the series considered is more relevant for our proposal than actual dating of peaks and troughs. Therefore, Figures 3 and 4 summarize these patterns for, on the one hand, IGAE as a reference series, and on the other, each of the remaining coincident or leading series.

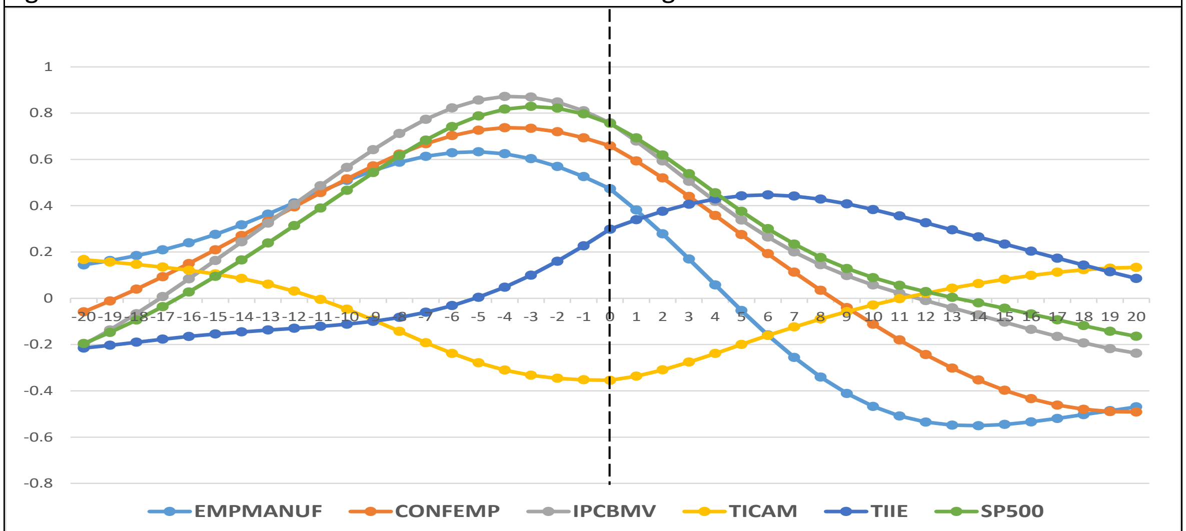
Fig. 3. Cross-correlation functions for IGAE and 5 other coincident series



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

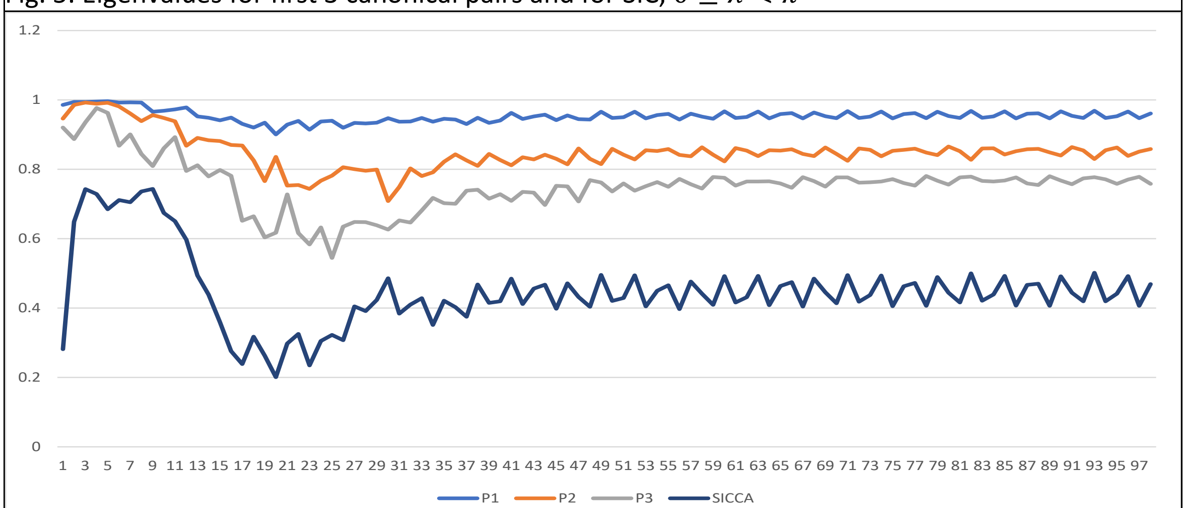
It becomes apparent from the first figure, that the series ACTIND, IMPORT, as well as TDOU are, on average, coincident series. It is not equally clear that IMSS, and INGXBYS are or remain in fact coincident. The first of these two shows a maximum correlation with IGAE of 0.8579 at a lag of three months; while the second shows a weaker relationship of around 0.68 with lags of 1, 2 and 3 months. Something similar occurs with the set of leading series among which, four of them EMPMANUF, IPC-BMV, CONFEMP and SP500, provide evidence of a leading behaviour with respect to IGAE, with maximum correlations from just over 0.6 to almost 0.9. Among the rest, TICAM would give the impression of being a coincident indicator, although with a weak contemporary correlation; TIIE appears to be lagging rather than leading the benchmark, also showing a weak correlation of almost 0.45 at a six-month lag.

Fig. 4. Cross-correlation functions for IGAE and 6 leading series



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Fig. 5. Eigenvalues for first 3 canonical pairs and for SIC, $0 \leq \lambda < \pi$



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

In turn, when our methodological proposal is applied to these series, six pairs of canonical series result. Additionally, we used INEGI leading and coincident composite indicators as input for the same methodology. Figure 5 shows the values of the squared norm of the coherence which, as has been pointed out, coincide with the eigenvalues from (4) for each of the harmonic frequencies $0 \leq \lambda < \pi$, for 3 of the 6 pairs of canonical series, as well as the only canonical pair between the SIC coincident and leading indicators. The first canonical pairs are therefore optimal in the frequency domain, in terms of the established criteria, over all the linear filters calculated from the coincident and leading series, including those considered in this work. It should be clear, however, that some of the remaining canonical pairs also contain useful information and are, by construction, almost orthogonal to that provided by the first pair. In other words, in the study of the cycle it is not necessary to restrict oneself to a single pair of indicators.

Fig. 6. First canonical pair, composite indicators. Coincident (C1) and Leading (A1).

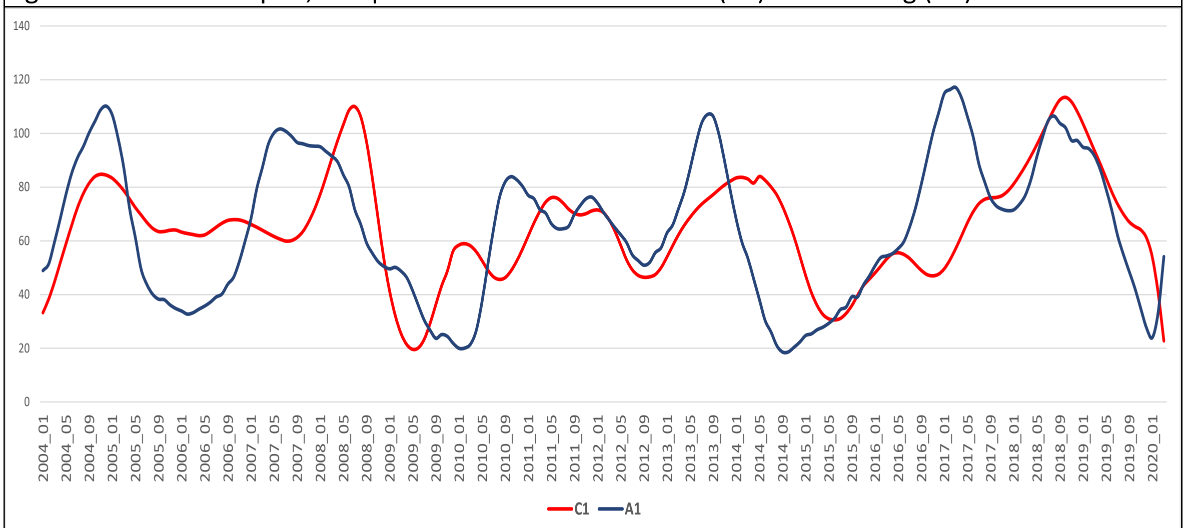


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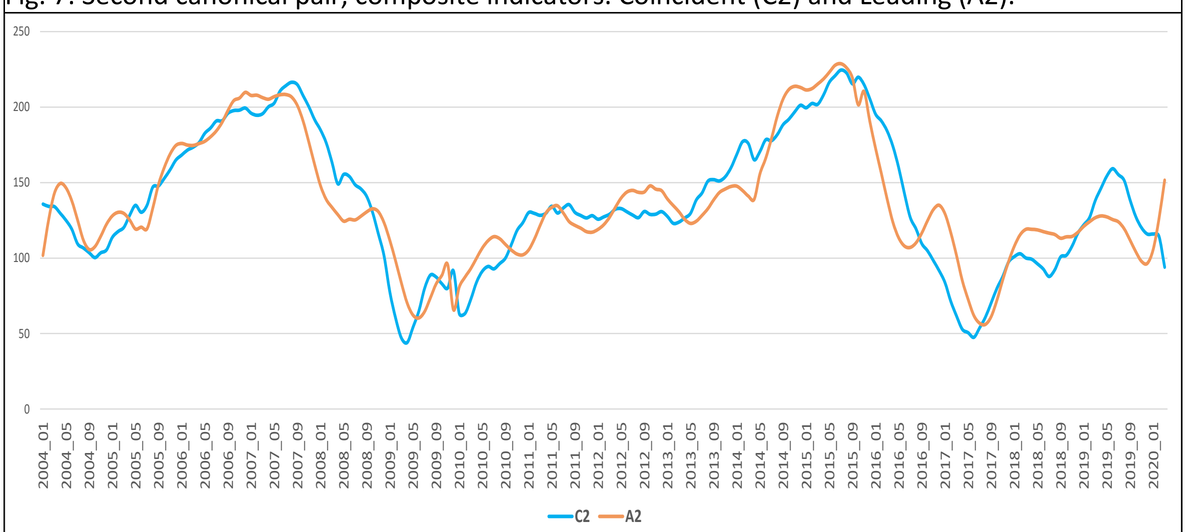
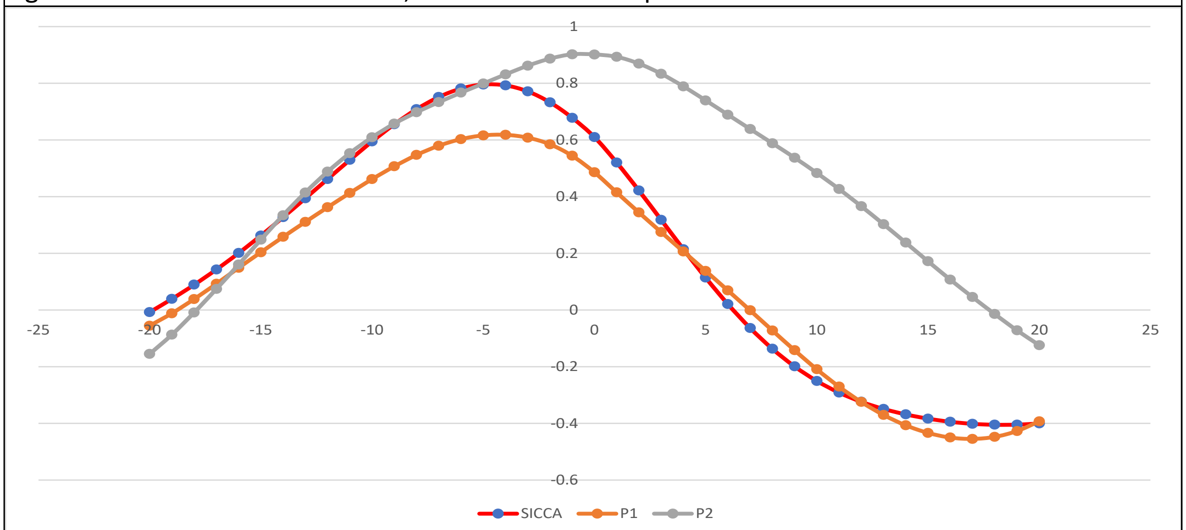
Fig. 7. Second canonical pair, composite indicators. Coincident (C2) and Leading (A2).



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

A part of the result of applying our proposal is shown in Figures 6 and 7, where the first and second canonical pairs are displayed, respectively. In the first case, the relationship between the coincident indicator and the reference series does not seem to be clear. Likewise, it is not entirely clear that the leading indicator actually leads the coincident one, or by how many months.
Regarding the second canonical pair, counterintuitively the new pair of indicators show a closer relationship. The leading indicator leads its counterpart at the beginning of the recovery periods (troughs), but the opposite occurs with the decline periods (peaks). It should be noted that the cyclical behaviour exhibited by this couple seems to refer to a longer-term cycle with an approximate duration of 5 years, which can only be roughly established given the length of the observation period.

Fig. 8. Cross-correlation functions, first two canonical pairs and SIC



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

The cross-correlation structures shown in figure 8 seem to confirm these findings. Indeed, the pattern for the first pair, denoted by P1, indicates a weak lead against what would be expected; the leading indicator appears to be ahead of the coincident, with a correlation of 0.62 at four to five months. Regarding the pattern corresponding to P2, the second canonical pair, we now have a clearer situation. Its maximum correlation at zero lag on average, which seems to confirm the alternation in leads described, reaches the value of 0.902, higher than that exhibited by the SIC pair. It is now evident that the optimality of the criterion in the frequency domain does not always translate into optimal results in the time domain.

Complementation of the proposal

The previous and unsatisfactory results have led us to put forward some hypotheses which we hope will lead to more favourable situations. The first of them refers to the fulfilment of various assumptions. In particular, the one that refers to the data. In a very simple way, it is possible to summarize it as "the coinciding series coincide with the reference indicator, and the leading ones lead it". Figures 1 and 2 tell us that, in terms of the second moments that are central to the procedure, not all the selected series seem to satisfy this assumption. This forces us to take a more careful look at our inputs. In addition to their relationship with the reference series, we are interested in finding out what the relationship between the coincident series with the leading ones is. To this aim, we propose to apply the proposed procedure to the case in which the set of coincident series is formed by only one of them at a time; that of the leading ones may contain the 6 original series, or some subset of them. We will denote the first case by 1X6. In other words, it is about anticipating future values of an indicator from a combination of present and past values of another set of indicators.

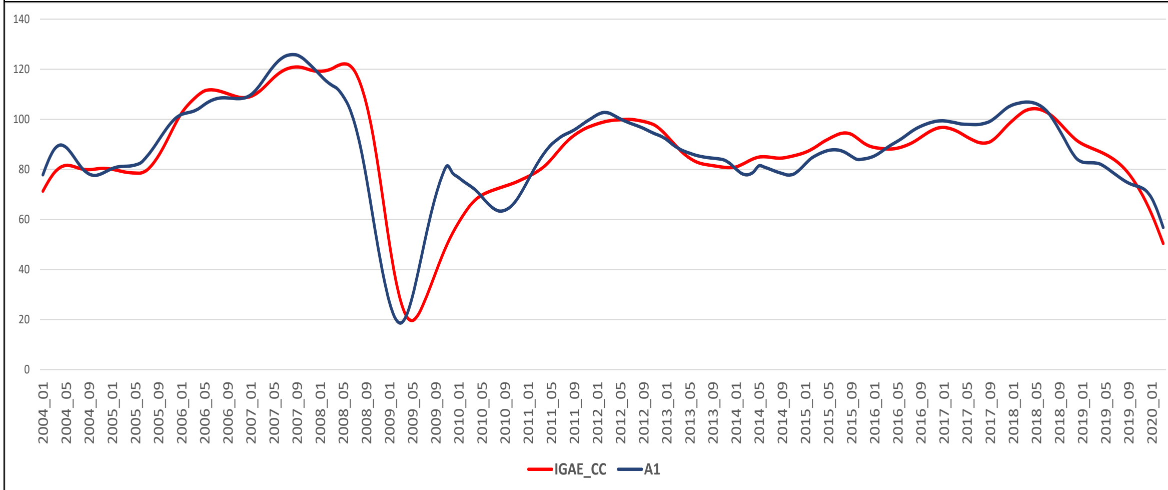
This exercise is reminiscent of the so-called Timely Indicator of Economic Activity (IOAE) [5], recently released by INEGI. In this exercise, an empirical nowcasting model is used to estimate both the annual percentage change and the levels of the IGAE and two large economic activities. The model uses "a wide set of timely and high-frequency indicators, for example: variables obtained from economic surveys, financial time series, indicators extracted from non-traditional sources, among others." In a strict sense, the indicator does not forecast the IGAE, but is an estimate for this indicator obtained from information whose publication takes place in a timelier manner and corresponds to the same month for which the value of the IGAE is estimated, which will be published subsequently.

Figure 9 shows the comparison over time of the evolution of each of the coincident indicators with the composite indicator from obtained from the 6 leading series. In each of the six panels that make up this figure, the coincident indicator whose behaviour is to be anticipated appears in red, while the corresponding leading indicator is shown in blue. Before resorting to other statistics to evaluate the result of these adjustments, it is already apparent to the naked eye that for the three coincident series IGAE, ACTIND, and IMPORT, a leading indicator with the desirable characteristics is obtained; The trends are closely followed, but showing a lead, particularly in the ridges and valleys, of at least a couple of months, overall. Similarly, the two series that in one way or another have to do with employment (IMSS, TDOU) do not exhibit an adequate behaviour. Even though smoothed versions of these two series and their corresponding leading indicators would show important similarities, the oscillations that some of them show suggest the difficulty of obtaining good forecasts of one from the other. At the other extreme is the case of the INGXBYS series for which the leading indicator appears to be, in fact, lagging behind.

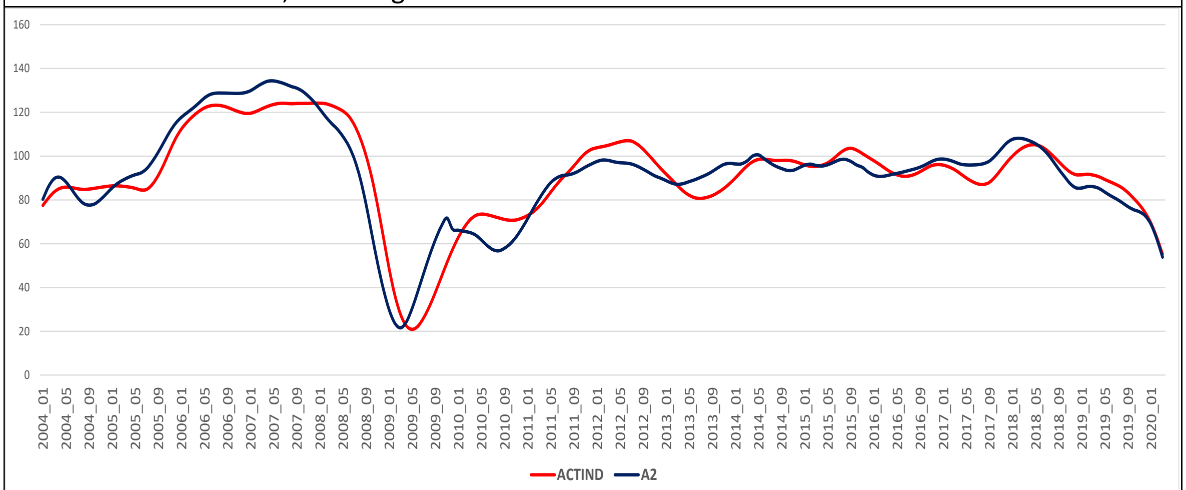
[5] https://www.inegi.org.mx/investigacion/ioae/

Fig. 9. Coincident and leading indicators, exercises 1X6. (Cont.)

9.1 Coincident: IGAE; 6 leading series



9.2 Coincident: ACTIND; 6 leading series



9.3 Coincident: INGXBYS; 6 leading series

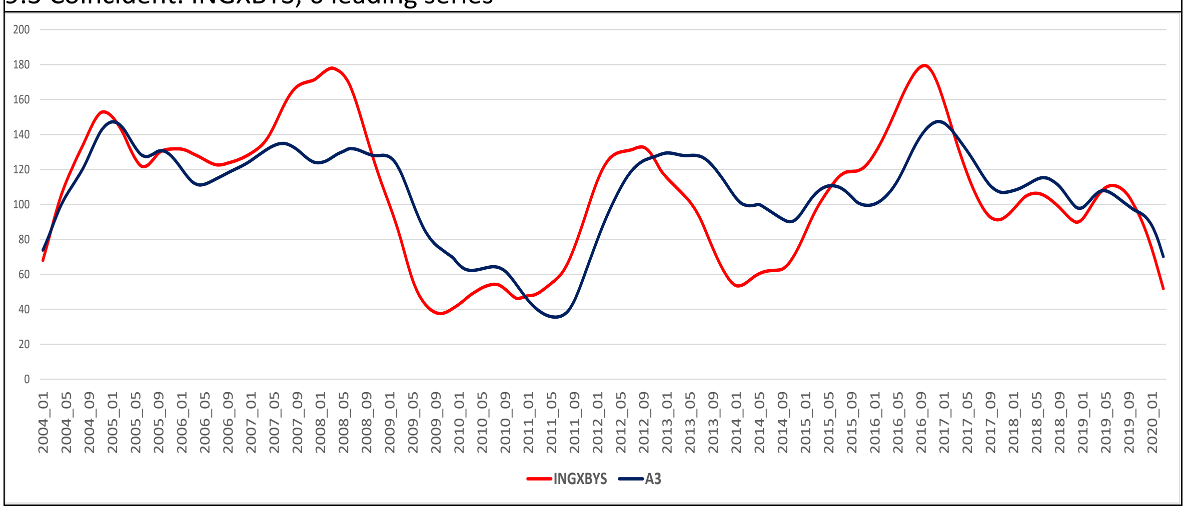
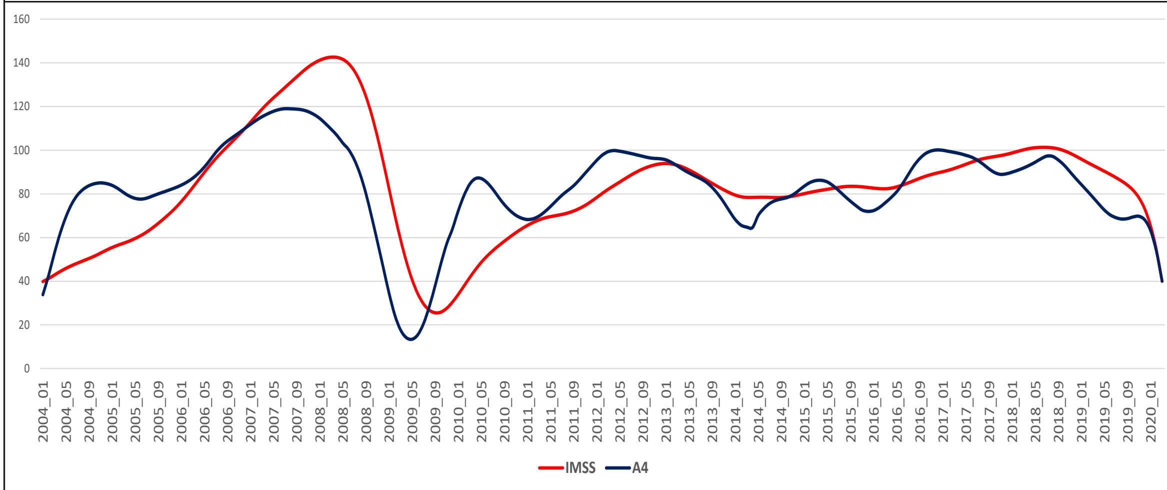
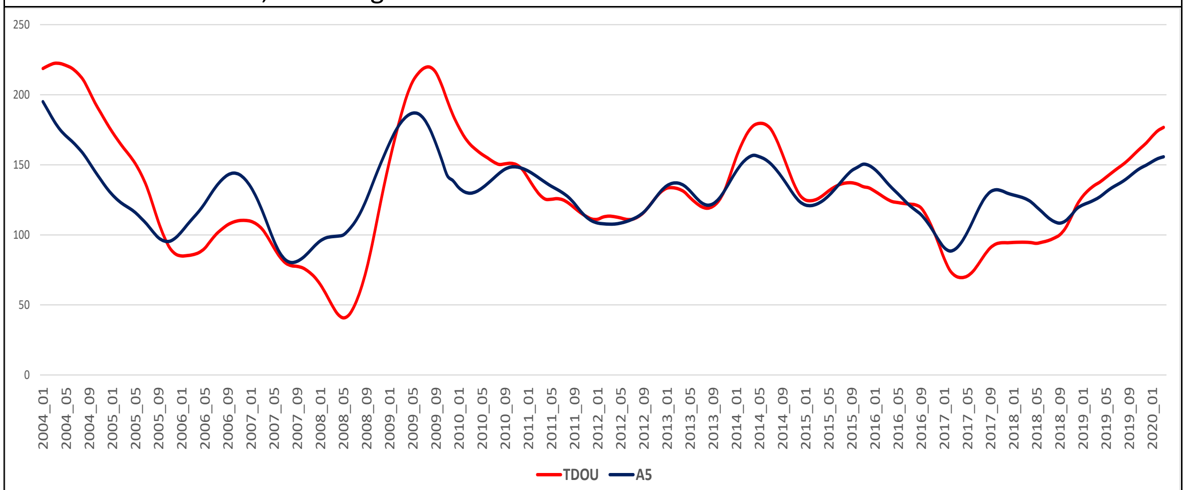


Fig. 9. Coincident and leading indicators, exercises 1X6. (Conc.)

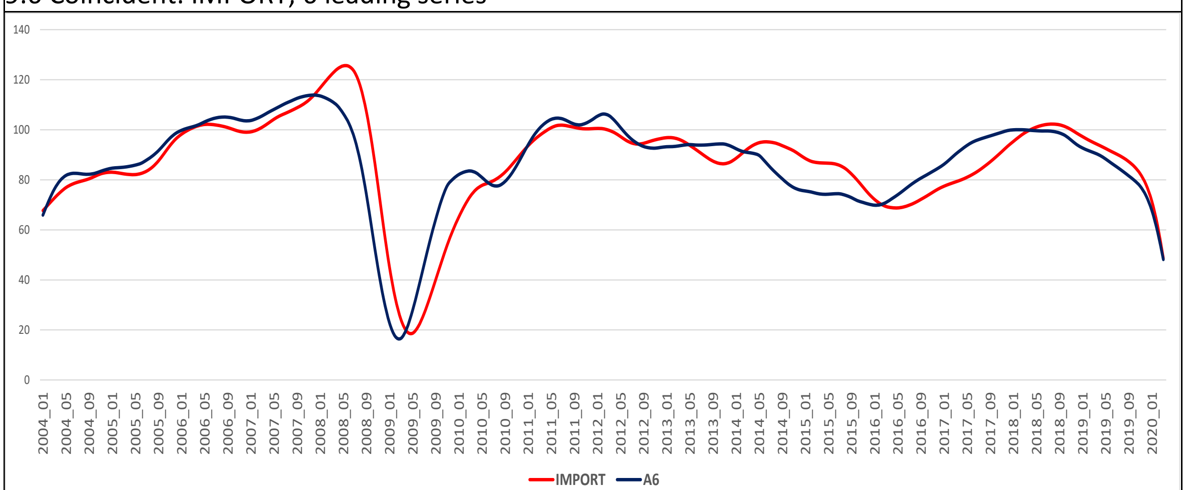
9.4 Coincident: IMSS; 6 leading series



9.5 Coincident: TDOU; 6 leading series



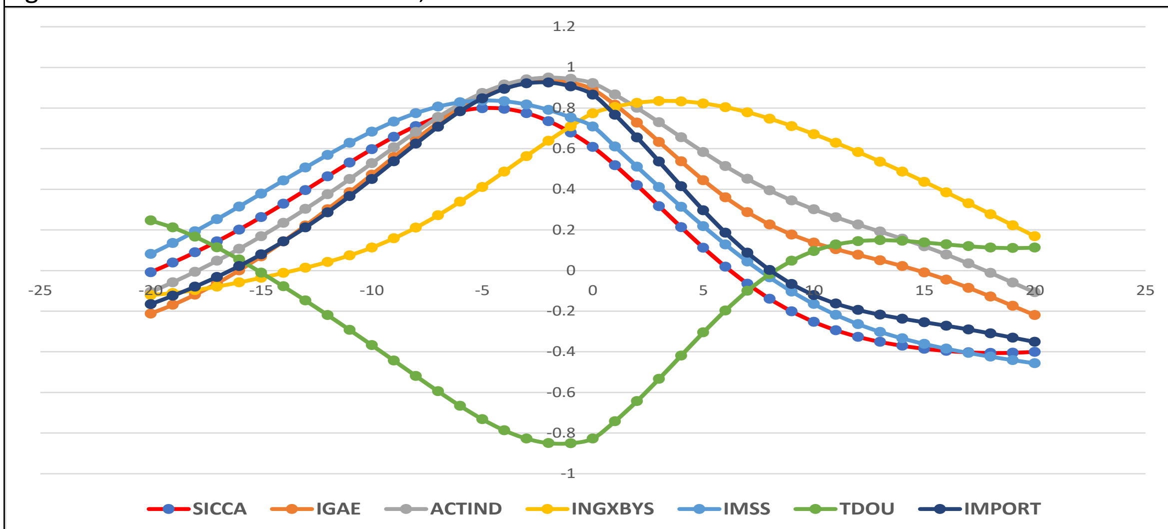
9.6 Coincident: IMPORT; 6 leading series



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Figure 10 summarizes all the cross-correlation structures for each of the above exercises; For comparison purposes, the one for SIC is also included. We find evidence that confirms some of our previous findings. For the IGAE, ACTIND and IMPORT series, the three pairs formed show a maximum correlation greater than 0.9 for a lag of two months. Both for these series and for the so-called IMSS, the maximum correlations are higher in magnitude than our benchmark, the SIC, whose maximum correlation with a lag of five months does not exceed the value of 0.8. For this fourth series, the maximum correlation with its leading indicator occurs for a lag of five months. It is, in fact, the series for which we have the longest lead. In the case of the TDOU series, on the other hand, the most highly correlated lead occurs at a lag of only one month, with a value of -0.85, which is still better than the benchmark. Finally, it is confirmed that the INGXBYS series is ahead of the leading indicator that seeks to anticipate it.

Fig. 10. Cross-correlation functions, 6 1X6 exercises.



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Now, if the general aim of our exercise is understood as obtaining good approximations for future values of one or more series from the present and lagged values of others, it is worth introducing a criterion that allows us to determine the success or not with which this purpose is achieved. This criterion, of course, must take into account the sums of squares of the approximation errors that result for each model and each lag. However, since the scales with which the SIC results are presented are different from those obtained from the application of our proposal, we decided to standardize the mentioned sums of squares by dividing them by the sum of the variances of each series[6].

$$SC_k = \frac{\sum_{t=1}^{T-k}(IA_t - IC_{t+k})^2}{Var(IA_t)+Var(IC_t)}, \quad (5)$$

where IA_t stands for the leading indicator and IC_t, for the coincident.

Table 2 summarizes the values of the previous criterion, both for the SIC and for each of the 1X6 exercises mentioned above, for each of the first 10 lags. In its first column it shows what happens with the SIC, reaching its minimum value for a lag of five months; It should be noted that said minimum value is above 50. Regarding the coincident series, among all of them the poor performance of the forecast that seeks to approximate future values for the TDOU series stands out

[6] Standardizations were attempted that considering correlations at different lags in the denominator. They were unsatisfactory since they presented decreasing trends from which it is not possible to identify the minimum value of the criterion, in a significant number of cases.

with values beyond 300; the criterion values included show a tendency to decrease, but for the lags shown it is not perceptible that a minimum value is reached. Nor does the forecast for the INGXBYS series show a desirable behaviour, which, as will be recalled, seems to be ahead of its leading indicator; perhaps this is the reason why they show a decreasing trend towards negative lag values. Consistent with the cross-correlation patterns shown in figure 10, the lowest value of the criterion is located at a 5-month lag. Now, contrary to what happens in the case of the SIC, this value is already below 50. In this way we come to identify the three series with the best values for the forecast error: IGAE, ACTIND and IMPORT. For the first two, the optimal values occur at a 2-month lag, while for the third there is almost a tie for the 2 and 3-month lags. It is worth noting, however, that the values of the five-month lag criterion are all below 50 for these series as well as for the IMSS series.

Table 2. Standardized sum of squares of forecasting residuals, 6 1X6 exercises.

LEAD = k	SIC	IGAE	ACTIND	INGXBYS	IMSS	TDOU	IMPORT
0	91.28	21.33	15.41	**61.71**	69.06	482.46	26.97
1	77.72	12.85	9.46	71.99	57.06	477.32	16.16
2	66.86	**8.87**	**7.14**	84.24	47.50	468.08	10.92
3	58.87	9.36	8.34	97.60	40.22	455.14	**10.87**
4	54.02	14.10	12.89	111.34	35.31	439.31	15.54
5	**52.50**	22.64	20.52	124.86	**32.85**	421.35	24.30
6	54.28	34.36	30.81	137.72	32.89	402.03	36.40
7	59.07	48.49	43.18	149.48	35.32	382.20	50.97
8	66.43	64.21	57.02	159.87	39.93	362.48	67.10
9	75.84	80.78	71.72	168.65	46.43	343.55	83.96

Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Table 3. Standardized sum of squares of forecasting residuals, 6 1X5 exercises, TIIE removed.

LEAD = k	SIC	IGAE	ACTIND	INGXBYS	IMSS	TDOU	IMPORT
0	91.28	32.48	17.56	33.26	53.07	505.11	42.32
1	77.72	21.68	13.03	**32.26**	41.19	502.88	30.93
2	66.86	15.27	**12.12**	34.46	31.97	496.58	24.36
3	58.87	**13.56**	14.75	39.37	25.45	486.21	**22.47**
4	54.02	16.49	20.83	46.47	21.65	472.20	25.09
5	**52.50**	23.69	30.05	55.21	**20.57**	455.14	31.82
6	54.28	34.50	41.90	65.11	22.16	435.85	42.06
7	59.07	48.08	55.71	75.65	26.26	415.39	55.00
8	66.43	63.60	70.73	86.44	32.58	394.78	69.76
9	75.84	80.30	86.30	97.12	40.77	**375.10**	85.58

Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

From the previous discussion, the coincident series INGXBYS and TDOU are candidates to exit the analysis, at least when the set of leading indicators considers the 6 series thus identified. Similar conclusions are reached when the TIIE and TICAM are excluded in succession from among the leading series (Tables 3 and 4, respectively). Based on these, it should be noted that the values of the criterion related to forecast residuals seem to grow as the size of the set of leading series decreases.

Table 4. Standardized sum of squares of forecasting residuals, 6 1X4 exercises, TIIE and TICAM removed.

LEAD = k	SIC	IGAE	ACTIND	INGXBYS	IMSS	TDOU	IMPORT
0	91.28	40.57	29.81	110.59	71.19	487.58	33.02
1	77.72	30.27	22.74	99.84	60.13	486.21	23.39
2	66.86	23.88	18.85	90.48	51.52	481.23	18.03
3	58.87	**21.74**	**18.17**	82.20	45.42	472.63	**16.87**
4	54.02	23.82	20.75	74.69	41.83	460.71	19.70
5	**52.50**	29.72	26.42	67.91	**40.73**	445.88	26.11
6	54.28	38.85	34.77	62.03	42.06	428.71	35.55
7	59.07	50.46	45.20	57.16	45.67	410.03	47.39
8	66.43	63.86	57.13	53.42	51.32	390.70	60.97
9	75.84	78.43	70.08	**50.68**	58.66	**371.72**	75.68

Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Despite the above, for the exercises that simultaneously contemplate these 4 coincident series, the forecast residuals tend to be smaller in magnitude as the number of leading series taken into consideration is reduced. Table 5 shows this behaviour.

Table 5. Standardized sum of squares of forecasting residuals, exercises 4X4, 4X5 y 4X6.

LEAD = k	SIC	4X4	4X5	4X6
0	91.28	51.54	115.94	201.12
1	77.72	38.34	102.23	192.63
2	66.86	30.85	91.85	185.09
3	58.87	**28.98**	84.75	178.62
4	54.02	32.26	80.93	172.87
5	**52.50**	39.96	**80.36**	167.48
6	54.28	51.17	82.66	162.28
7	59.07	64.86	87.58	157.24
8	66.43	80.10	95.07	152.58
9	75.84	96.04	105.01	**148.52**

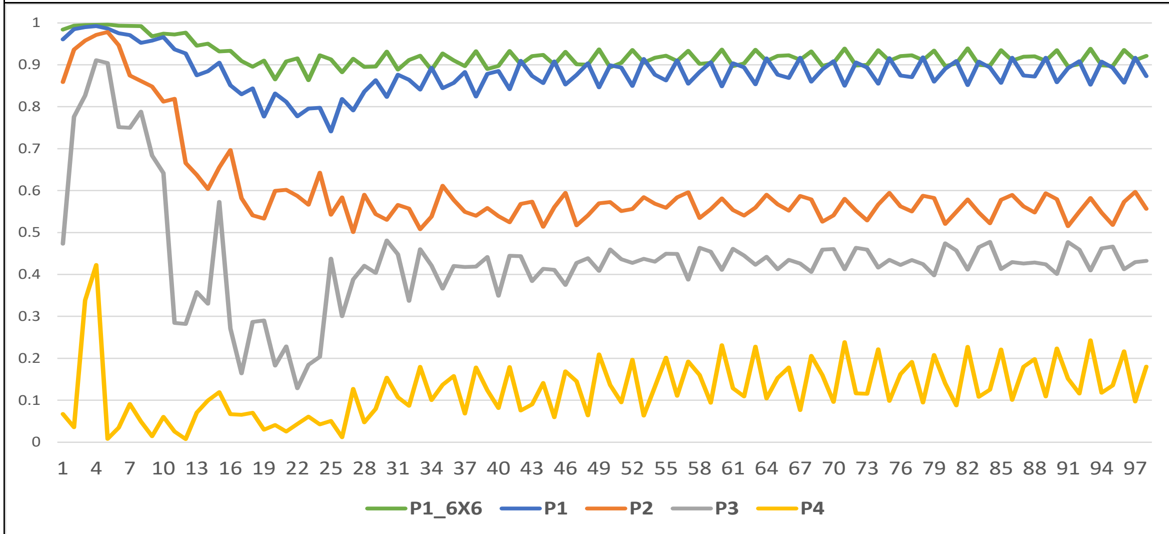
Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Proposal for the system of coincident and leading indicators.

From the results discussed in the previous section, we conclude that the best results are obtained when two sets of series are selected, each consisting of four of them. The 4X4 exercise that we will present next considers the 4 series IGAE, ACTIND, IMSS, and IMPORT as the set of coincident ones; on the other hand, among the leading ones we include the EMPMANUF, CONFEMP, IPCBMV and SP500 series.

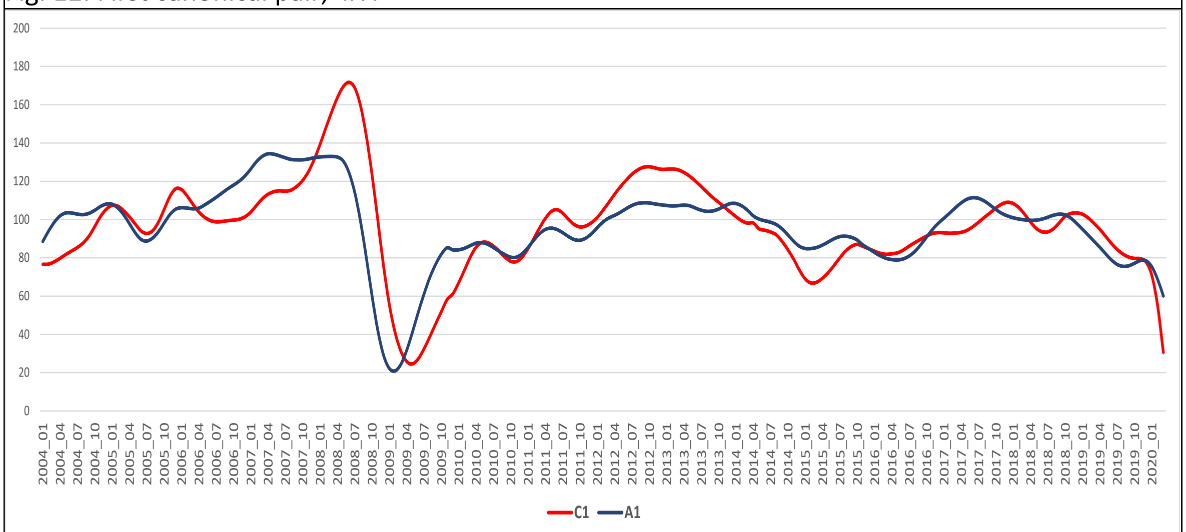
We observe from figure 11 that the elimination of information gives rise to a slight decrease in the values of the square of the norm of coherence for the first canonical pair at different frequencies. In other words, in terms of this criterion, the results presented would be considered sub-optimal. The loss is most noticeable for a set of cycles with a duration of between 7 and 20 months, approximately. In general, the remaining canonical pairs show values that are far from optimal behaviour. In other words, for this example it is unlikely that they contribute significantly to the study of the economic cycle.

Fig. 11. Eigenvalues for P1-P4 in 4X4 exercise and for first canonical pair of 6X6, P1_6X6.



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Fig. 12. First canonical pair, 4X4

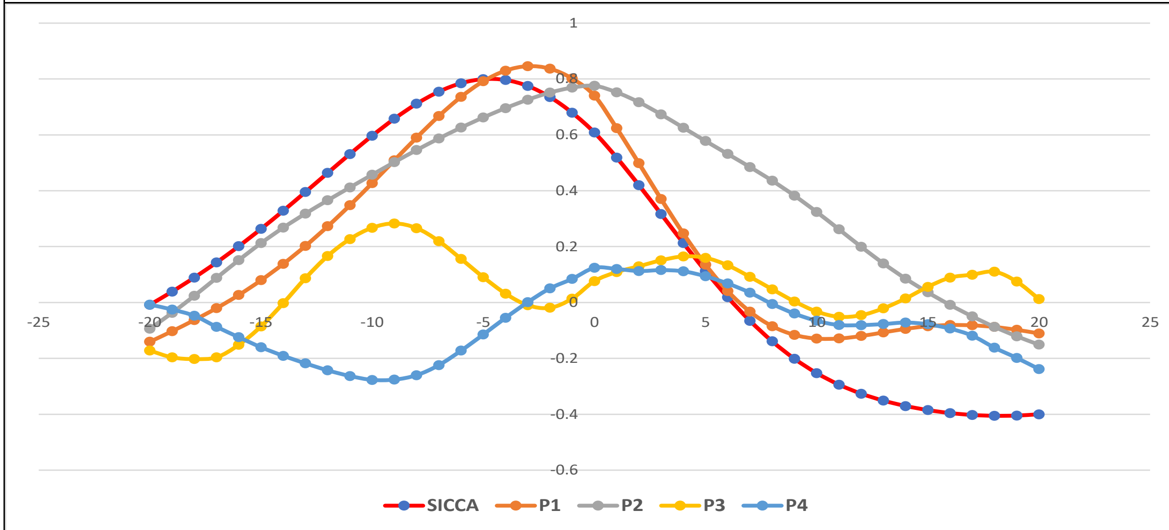


Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

The joint evolution of the coincident and leading indicators, that is, of the elements of the first canonical pair, is shown in figure 12. As can be seen, the relationship between both indicators is close to what is expected of them. Of course, their desirable behaviour between 2008 and 2009 stands out. Towards the end of the period, both show a decreasing trend that begins in October 2018, for the leading indicator, while the coincident begins its decline between December of that year and January of the following year.

The correlation structures for the four canonical pairs obtained in this exercise, as well as for the SIC, included for comparison purposes, are shown in figure 13. The weak linear relationship between the elements of the second, third and fourth canonical pairs is evident. This time, however, the association between the elements of the first pair reaches a maximum value close to 0.85 at a three-month lag, which is higher than the SIC benchmark throughout the lags shown, all of them lower than 0.8.

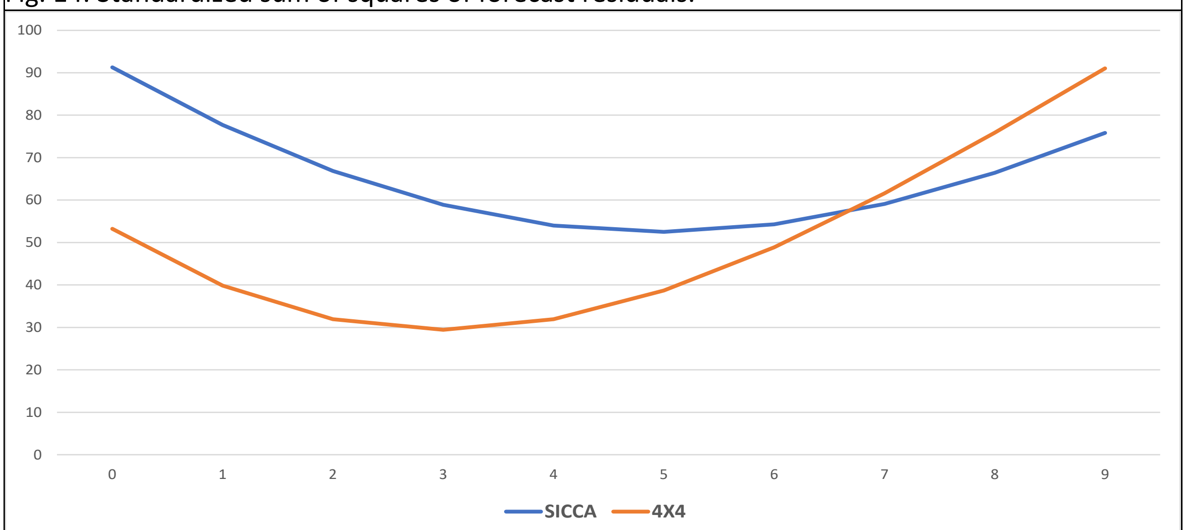
Fig. 13. Cross-corelation structures for SIC and 4 canonical pairs



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Regarding the criterion regarding the forecasting capacity of the leading indicator in relation to the coincident, it is also found that the new system of indicators is superior to the SIC, with an optimal three-month lead. Only as we move away from the respective optimum is it clear that our proposal is not uniformly better.

Fig. 14. Standardized sum of squares of forecast residuals.



Source: own calculations from database in https://www.inegi.org.mx/app/reloj/#

Optimality in the frequency domain does not automatically translate into optimal cross-correlations.

When we discussed the limitations of the proposed procedure, shown in Exercise 6X6, we referred to more than one hypothesis to explain them. So far, we have focused on data and its limitations. However, we know a second cause for which, however, we do not yet have an adequate answer. It is to be remembered that our proposal proceeds by solving an optimization problem, frequency by frequency. As we have already indicated, the optimization for each frequency does not seem to necessarily lead to optimal behaviours in the aggregate represented by the Fourier inverses. This time the suspicion falls on the same eigenvectors that are the solution to each optimization problem.

The eigenvectors are unique, except for their multiplication by a complex of norm one. It is easy to see that for the Hermitian matrix $\Phi_X(\lambda)$ it must be held that, in view of the commutativity of the product of diagonal matrices, equalities in (6) are satisfied. In them, the columns of the matrix $B(\lambda)$ are the corresponding characteristic vectors, the diagonal matrix $\Delta(\lambda)$ is formed by the characteristic values, and $\mathrm{E}(\lambda)$ is an arbitrary diagonal matrix, whose non-zero elements have the form $e^{i\theta_j(\lambda)}$.

$$\Phi_X(\lambda) = B(\lambda)\Delta(\lambda)\bar{B}^t(\lambda) = B(\lambda)\mathrm{E}(\lambda)\Delta(\lambda)\bar{\mathrm{E}}^t(\lambda)\bar{B}^t(\lambda). \quad (6)$$

Said expression gives rise to equivalent spectral representations for the matrix $\Phi_X(\lambda)$. However, when these characteristic vectors are used differently, as in the case at hand, the results may not be as favourable. It is not clear that the pair of canonical series $\zeta_j(t)$ and $\eta_j(t)$, defined in (7.1) and in (7.2), turn out to be invariant when the phases for each frequency can be modified on a whim.

$$\zeta_j(t) = \int_0^{2\pi} \overline{A_j(\lambda)}^t e^{i\lambda t} dZ_X(\lambda) \neq \int_0^{2\pi} \overline{A_j(\lambda)}^t e^{i\left(\lambda t - \theta_j(\lambda)\right)} dZ_X(\lambda) \quad (7.1)$$

$$\eta_j(t) = \int_0^{2\pi} \overline{B_j(\lambda)}^t e^{i\lambda t} dZ_Y(\lambda) \neq \int_0^{2\pi} \overline{B_j(\lambda)}^t e^{i\left(\lambda t - \theta_j(\lambda)\right)} dZ_Y(\lambda) \quad (7.2)$$

In our attempt to translate the routines into the R language we have found that the results can be very different, perhaps for the same reason. The previously discussed results were obtained using the DEVCHF routine for the eigenanalysis of complex Hermitian matrices, part of the IMSL library, originally written in Fortran. This routine results in orthonormal vectors and, apparently, imposes the restriction that the input with the highest norm of the eigenvector is forced to take a real value. In the case of R, we have used the EIGEN function which, in addition to the orthonormality condition, forces the first of the components of each vector to be a real number. In both cases, the operation is equivalent to multiplying a vector by a complex on the unit circumference. This seems to be the main reason for the large differences between the results.

Final comments and conclusions

We have shown an application of the procedure described by Brillinger, to the problem of developing optimal weights for a Cyclic Indicator System. From our discussion, optimality in the frequency domain is not directly inherited by time-domain indicators. We have also described how to take advantage of the same methodology to select, from among a group of candidate indicators, those which will become part of the final proposal. Despite the above, we arrive at composite indicators that compare favourably with the proposals currently in operation, based on a set of desirable criteria. Consequently, the procedure must be replaced by suboptimal solutions for the time being, until the conditions are found that lead from the optimal solution in the frequency domain to optimal results in the time domain. These conditions will have to be reflected in proposals for modifying routines for the numerical calculation of characteristic values and vectors. We are exploring some promising alternatives that we hope to be able to report soon.

The path followed to achieve the presented results illustrates a way to select the series to integrate each set. Indeed, even when the peaks and valleys shown by a series anticipate or coincide with those of the reference series, it seems useful to consider other aspects such as those discussed here before deciding. On the other hand, in conditions of high volatility of the indicators, it seems necessary to review the exclusion of some series or the inclusion of others.

Our presentation has focused on econometric and statistical aspects of cyclical indicator systems. We do not know what the SIC results would be if a similar selection of the input data were made at this time. There seems to be an awareness that some of the indicators considered by this system have lost, in the recent past, their coincident or leading character. However, no action has been taken to eliminate them or replace them with others. This does not appear to be due to the desirability of maintaining comparability both over time and with other countries. In the first case, the elimination of the trend and smoothing using the Hodrick-Prescott filter leads to changes in the values of the indicators even in the remote past, month by month. That is, new figures are not comparable with those obtained in the past. However, since every month the same procedure is applied throughout the whole observation period, comparability over time within the new results occurs, and this seems reasonable. In the second, since the selection of the basic indicators is carried out independently in each country, comparability does not seem to be a concern.

References

Box, G. E. P., y Jenkins, W. (1970), Time Series Analysis: Forecasting and Control**,** Holden-Day, San Francisco, 1970.

Brillinger, D. R., Time Series, Data Analysis and Theory, Holden Day, Inc., San Francisco, 1981.

Burns, A. F., y Mitchell, W., Measuring Business Cycles, National Bureau of Economic Research. Studies in business cycles**,** First Edition (1946). Retrieved on Nov. 11, 2020 from https://www.nber.org/booksbyseries/SBC.html

Bustos (1993), Statistical Methodology for Leading Indicator Systems, Bulletin of the International Statistical Institute, Contributed Papers, Book I, pp. 189-190, 49th session, Florence, Italy.

The Conference Board (2001), Business Cycle Indicators Handbook, The Conference Board, 2001. Retrieved on Nov. 11, 2020 from https://www.conference-board.org/publications/publicationdetail.cfm?publicationid=852

The Conference Board (2020), U.S. Business Cycle Indicators, THE CONFERENCE BOARD LEADING ECONOMIC INDEX (LEI) FOR THE UNITED STATES AND RELATED COMPOSITE ECONOMIC INDEXES FOR SEPTEMBER 2020. Retrieved on Nov. 11, 2020 from https://conference-board.org/pdf_free/press/US%20LEI%20-%20Tech%20Notes%20OCTOBER%202020.pdf

Hannan, E. J. (1970), Multiple Time Series, John Wiley & Sons, Inc., New York.

Hodrick, R. J. and Prescott, E. C., Postwar U.S. Business Cycles: An Empirical Investigation, Journal of Money, Credit and Banking, Vol. 29, No. 1 (Feb., 1997), pp. 1-16, Blackwell Publishing.

INEGI (2015), Metodología para la construcción del sistema de indicadores cíclicos, Instituto Nacional de Estadística y Geografía. -- 2ª. ed. -- México: INEGI, 2015. Retrieved on Nov. 11, 2020 from http://internet.contenidos.inegi.org.mx/contenidos/productos/prod_serv/contenidos/espanol/bvinegi/productos/nueva_estruc/702825073961.pdf

Mitchell, W. and Burns, A. F. (1938), "Statistical indicators of cyclical revivals", Bulletin 39, NBER, New York. Retrieved on Nov. 11, 2020 from https://www.nber.org/books-and-chapters/statistical-indicators-cyclical-revivals/statistical-indicators-cyclical-revivals

Moore, G., Leading Economic Indicators: New Approaches and Forecasting Records Lahiri and Moore (Eds.), 1993

OCDE (2012), OECD System of Composite Leading Indicators, OECD, Paris, 2012. Retrieved on Nov. 11, 2020 from http://www.oecd.org/sdd/leading-indicators/41629509.pdf

Pollock, D.S.G., Econometric Filters, Working Paper No. 14/07, March 2014, University of Leicester, UK.

Pollock, D.S.G., Trends, Cycles and Seasons: Econometric Methods of Signal Extraction, Working Paper No. 14/04, February 2014, University of Leicester, UK.

Pollock, D.S.G., Filters, Waves and Spectra. *Econometrics*, *6*, 35, 2018. Retrieved on Nov. 11, 2020 from https://www.mdpi.com/2225-1146/6/3/35

Stock, J. H., Watson, M. W., New Indexes of Coincident and Leading Economic Indicators, en NBER Macroeconomics Annual 1989, Volume 4, Blanchard and Fischer (eds.), MIT Press.

Yabuta (2010)), New tools for tracking the Mexican Business Cycle, ESA/STAT/AC. 223/S3.5, Third International Seminar on Early Warning and Business Cycle Indicators, 17 – 19 November 2010, Moscow, Russian Federation. Retrieved on Nov. 11, 2020 from https://unstats.un.org/unsd/nationalaccount/workshops/2010/moscow/AC223-S35.PDF