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Reasoning with Risk: Teaching probability in the time after COVID

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Abstract:
Covid has emerged as the most deadly virus in this century. Time will judge the world's response but the pandemic presents a valuable opportunity in education. .Risk is a key aspect of life and probability is the mathematical tool to address risk. Our aim is to explain how risk should be treated in education. Decisions in risky situations are linked to the various approaches to probability and to rational and behavioural approaches to decisions. Faced with the twin character of probability and risk, we argue that these concepts should be developed together in teaching. The conceptualisation of probability in terms of APT (a priori theory), FQT (frequentist theory) and SJT (subjectivist theory) forms the background to classify the pertinent constituents of the arguments

Keywords:
Probabilistic thinking, Uncertainty; Decision theory; Subjectivist probability

1. Introduction:

The title of this paper alludes to the origins of probability in the midst of time, yet is highly relevant in 2020, the year of the perceived high risk and devastating cost of covid 19 (C19). We deal with the nature of chance, as well as probabilistic knowledge and reasoning. Risk is the precursor of probability.

We assert that the teaching of probability does have fundamental problems partly linked to emotional and psychological considerations. This is demonstrated by our own extensive research (based on historical studies as well as in-depth interviews) and that of Kahneman and Tversky and many others. This research has been influential in the proceedings of ICME and ICoTS conferences on probability over the last few decades. The present research is an extension of work of Borovcnik and Kapadia (2018), with new insights from the covid-19 pandemic, which is clearly an issue of risk and risk handling.

2. Methodology:

Our aim is to present a rationale for supporting the teaching of probability as a means of addressing risk in the curriculum. We argue that modern views on risk are a new justification for the teaching of probability.

While most applications use probability in an SJT (subjectivist theory) connotation, the concepts are introduced in school by artificial experiments that embed the ideas within a pure FQT (frequentist theory) meaning, alongside an APT (a priori or equiprobable theory) setting that allows direct calculations for probabilities in simple cases of equal likelihood. We use the terminology of these acronyms (APT, FQT, SJT) introduced in Borovcnik and Kapadia (2014) to delineate the key strands, which are important for the teaching of probability. There is an axiomatic setting for these three approaches:

- a reformulation of Laplace (1951/1812) for APT;
- the direct characterisation of FQT by von Mises (1919);
- the indirect theory for FQT by Kolmogorov (1956/1933); and
- de Finetti's axiomatization of preferences (1992/1937) for SJT.

Yet the historic evolution has been – and still is – signified by fierce controversies (Hacking 1975; von Plato 1994). Our systematic notation was inspired by Çınlar (2011). We assert that our terminology encapsulates in a direct and dynamic way the key issues, which are under discussion within the field of probability education.

Two other terms are used in this paper, adopted from the work of Spiegelhalter who has developed the idea of "micromort" and "microlife" to describe very low probabilities. A micromort is an event, which has a 1 in a million chance of leading to death. A microlife is 30-minute period within the average life of a young person aged 30, which is around 1,000,000 half hours: a particular event may increase or reduce a life by 1 microlife. For example, drinking one unit of red wine daily is deemed to increase life by 1 microlife, while eating a portion of red meat reduces life by 1 microlife, on average of course (Blastland & Spiegelhalter 2013).

For probability education, we strongly promote one of aims of the Schools Council Project on Statistical Education (1980, p. 27),

> "Children should become aware of, and appreciate the role of statistics in society ... and that the concept of risk is closely related to and dependent upon probability."

3. Result:

We start from the following definition of risk (Borovcnik & Kapadia 2011a, p. 1). 'By risk we understand a situation with inherent uncertainty about the (future) outcomes, which are related to impact (cost, damage, or benefit).'

Does it pay to accept the risky situation or is it better to do nothing? To take out an insurance policy is a risky situation with reversed signs: do nothing means accept the risk, buy the policy means avoid the risk and make a certain payment. Games and insurance were the two driving forces for the emergence of the concepts in the history of probability (Maistrov 1974, p. 5). With this setting, we see that many situations of probability are naturally embedded in the context of risk.

Hansson (2007, p. 2) is more detailed and distinguishes five definitions of risk:

(1) An *unwanted event* which may or may not occur; […]
(2) the *cause* of an unwanted event which may or may not occur; […]
(3) the *probability* of an unwanted event which may or may not occur; […]
(4) the statistical *expectation value* of an unwanted event which may or may not occur; […]
(5) the fact that a decision is made under conditions of *known [rather than unknown] probabilities ("decision under risk") as opposed to decision under uncertainty*.

The way that risk is perceived meets the need to derive one figure as a characteristic of risk inherent to a choice: it uses both the impact (which could be measured by money or utility) and the probability of the various outcomes of the risky situation. It attributes an expected value to the random variable of all possible outcomes that are related to an option. This facilitates the interpretation of a single risky situation but it also enables us to order several choices: i.e., to accept the risky situation or to do nothing; or to choose between several options of which all bear different consequences in terms of impact and its related probability. For example, the risk of taking out a comprehensive policy for a car can be compared to the alternative of taking no insurance.

In decision theory, when a null hypothesis is tested against an alternative hypothesis we have two types of errors involved: to reject the null hypothesis if it in fact applies (type I) and not to reject it if in fact the alternative hypothesis applies (type II). When we speak of the risk of wrong decisions, then we often use risk in the sense of (3) if we address the size of this risk (alpha or beta); however, we switch to the meaning (1) of risk if we only encompass the event (i.e., to commit an error of that type).

Once, a decision involves several stakeholders, the following question arises: Can a rational approach be pursued by all stakeholders equally, or do these personal judgements – still following a rational approach – lead to different decisions depending on their role in the decision? For example, it is arguable as to whether screening programmes for the early detection of cancer really do have a positive effect for every-one (Borovcnik & Kapadiia 2011). The benchmark for the rational approach to choice under uncertainty is the expected utility hypothesis, under which expected utility is maximised by rational players who follow the laws of probability.

Behavioural economists and psychologists in contrast contend that people often make decisions using simplistic or “fast and frugal” heuristics rather than using expected utility because of limited time, information, and cognitive capacity (Gigerenzer 2002). This behavioural view highlights psychological traits that violate the assumptions underlying the ‘rational’ view. In both approaches, the impact of a decision is measured in utility and not in money terms (Resnik 1987). Since assignments of utility and probability have to be made (which vary between people), it may be that people do not use that procedure to evaluate risk.

The rational assumption is that the brain learns over time to make the best decision, based on previous experiences. However, formal processing only occurs intermittently. Conversely, people are frequently incoherent in assessing, assigning and processing probabilities, even in highly artificial settings where information is complete, and they find it particularly difficult to assess probabilities which are very low or very high, partly because there is less scope for learning in these cases. People’s judgement is also influenced by the fact whether the situation is a winning or losing situation; Kahneman and Tversky (1979) have shown that in situations of gains, people seem to be risk-averse, in situations of loss, they seem to be risk-prone. They talk about heuristics and biases. The availably heuristic has played a major role with C19: almost all other risks in the world have been forgotten.

A key challenge for education is to understand how decisions are made. In terms of real-life examples, there are several topical issues from this century. Here we deal briefly with three examples: pandemics, the problems from ash clouds, and dealing with bovine spongiform encephalopathy (BSE). Common to these situations is that there is a threat of severe impact but little is known about the probability or risk of the occurrence of the adverse events. The underlying issues are quite deep and complex, which may be seen from the analysis. It is hard to disbelieve experts, but one should always remember that they have their own prejudices, not least relating to their self-perceived expertise (Borovcnik & Kapadia 2011a, 2011b).

With regards to pandemics, scares have arisen over the last few centuries and also in 2020. The scale of concern has increased markedly in the last few decades with increasing travel and enhanced communication, but it is not clear that the responses have improved in terms of a better understanding of probability. Indeed, it could be argued that a poor understanding of probability has led to a worse response politically, certainly in terms of the extra money spent in the prevention of spread in many countries. This trend is perhaps encouraged by pharmacological companies, who can make large profits from “scare-mongering” (Blastland & Spiegelhalter 2013).

With the C19 pandemic in 2020, some countries, notably in the Far East fared relatively well with carefully calibrated measures. Western countries have generally performed poorly, partly because of the age profile of the population, as well as difficulties in ensuring compliance in free societies with attention seeking and sensationalising news and media outlets. The risks from the virus took time to emerge but are still not well understood neither by the public nor by politicians. For politicians, the risk of over-stretched hospitals far out-weighed the risk of taking a measured approach, even though this led to massive expenditure, In It is not possible to accurately calculate the number of deaths if no measures, such as a lockdown, had been taken. An estimate could be made from patterns across the world. For systematic modelling, much more data would need to be processed.

In 2020, there have been about 80 000 (12%) excess deaths in UK (ONS 2021). In April 2021, the number of excess deaths fell and was 6% below the average for 2015-2019. This leads to our hypothesis that, in the main, C19 led to earlier deaths of many people by a year. Hence, we hypothesise that the extra deaths from the virus, *without lockdowns* would have been less than 20 000 in UK. It should be stressed that this estimate is rather crude. The economic cost is estimated to be about 8% of GDP in UK ($200 billion), i.e., $2×10^{11}$ This amounts to a cost of about $$10^7$ per excess death avoided by lockdowns. There are other factors such as serious health issues amongst some (long covid), as well as days lost for those who catch the virus. This would still be a cost of about $1 million per serious adverse event (death or serious illness) avoided. This scenario can provide an excellent forum to discuss risk and probability in the future, after the pandemic has been eliminated.

The ash-cloud in 2010 also had a significant adverse economic effect. Flights of aeroplanes were stopped and only resumed after great commercial pressure was applied by airlines. The BSE controversy in the United Kingdom led to mass culls, which some now believe were unnecessary; it might be that positive-tested BSE cattle were actually false positives (see Dubben & Beck-Bornholdt 2010, p. 64). Two underlying issues are the confusion between probabilities and impact, and the difficulties to update probabilities with new information. The confusion between probabilities and impact, as measured by utility arises when probability relates to everyday situations and insufficient comparable data is available so that FQT does not give a sufficient basis for an estimate of the probability of the event of interest.

Updating risks by new data involves Bayes formula. A relatively complex mathematical algorithm has to be applied to evaluate the new risk. To interpret the notions of false positives and positive predictive values in the context of medical diagnosis, FQT is of no help as relevant frequentist information is missing. Mathematical statements about conditional probabilities are quite sophisticated so that people tend to oversimplify matters. When there is a chance of a false positive for a disease, which affects a small proportion of the population most people, including doctors, grossly overestimate the positive predictive value, i.e., the probability of actually having the disease. The converse issue arises in a pandemic where false negatives enable people to avoid quarantine restrictions (some C19 tests have an efficacy of only 50% and relatively high false negatives). Thus for detecting a disease, it is important to reduce false positives, while to counter a pandemic, one should reduce false negatives. This is an important lesson to include in probability education.

The risks to life are often greatly exaggerated in the media, leading to public fears and concerns, with over-cautious responses from politicians. Even if the risk is very low, no politician can afford to 'do nothing' as his/her reputation would be irrevocably damaged if the highly unlikely event does indeed occur. For example, a dubious, haphazard, and overpriced purchase of breathing masks against swine flu (H1N1) by the Austrian Ministry of Health in 2005 led to a dispute about the rationality of the decision. The then Minister of Health in reply to a parliamentary query to him answered (Austrian Ministry of Health 2012, p. 2):

> "Based on the experience with aviation flue (H1N1) it is to state that the pressure of the media and the inconsistent advice of the experts have exerted a more than critical part in making the decision."

A major point to make is that risk is lower now than at any point in the past. Tragedies are far less common. Despite popular perceptions, life has actually become much less risky compared to even a century ago. For example, life expectancy has increased by 13 years for males between 1910 and 2010, while infant mortality is much reduced. That means that it is not a simple shift of 13 years in life-expectancy distribution but a drastic change of the shape of the lifetime distribution by the infant mortality decrease. Our main concern here is whether the underlying probability is well understood, and how a rational perception of the context and the evaluation of risks and probabilities can be improved; this needs to feature strongly in education.

A convincing model of probabilistic thinking still is an open research question as stated not only by Batanero & Borovcnik (2016). They delineate basic primary and secondary higher order categories of thinking and intuitions, with seven sub-categories respectively, which can be embedded in the underlying ideas in the context of risk.

1. The ability to discriminate between randomness and causality;
2. The ability to balance between psychological and formal elements of probability;
3. The understanding that direct criteria for success in probability are missing;
4. The understanding that criteria for reflecting on a random situation differ from those, which may be applied in selecting a decision;
5. The awareness of the theoretical character of probability (combining SJT, APT, and FQT aspects) including the case of small probabilities;
6. The awareness of conditional probability and its asymmetry;
7. The development of concepts building probabilistic evidence (such as probabilistic dependence as conceptualised by the correlation coefficient).

4. Discussion and Conclusion:

Probability can help to explore risky situations and make the final judgements more rational. The usual contract in games and in the insurance situation is signified by a swap of risks. One stakeholder who has no risk offers to the other to 'take over the risk' who then asks for monetary compensation. To fix the prices for such an exchange, it is essential to determine the probabilities of all (not only the adverse) outcomes – either by APT, FQT or SJT information and calculate an economic value of the diversity of outcomes. Similarly, probabilistic modelling is applied to derive the Black-Scholes equations and solve them to determine the price of futures at the stock market (see Hull 2009), which forms the basis for modern finance.

Historical struggles provide a further valuable orientation. While empirical research about how people think and how successful teaching programmes have been helpful to improve teaching plans, we should not lose sight of key concepts and strategies from the past. One key lesson from history is that probability has always been a *pluralistic* concept and has drawn its meaning from the interplay of its interpretations.

Finally, we comment on progress in probability education relating to ten assertions we made at the end of Chance Encounters (Kapadia & Borovcnik, 1991).

1. People use personal experience in assessing chance in a rather haphazard manner.
2. People process information in a rather incomplete way.
3. People process information in a way biased by memorable events.
4. People find it hard to assess probabilities, which are very low or very high.
5. People do not assign values of 0 for impossibility and 1 for certainty.
6. People equate certainty and impossibility with physical rather than logical events.
7. People equate 50-50 chances with coin tossing.
8. People assign equal likelihood in unknown situations.
9. People are incoherent in assigning and in processing probabilities.
10. People are supra-additive.

Assertions 1–3 relate to personal, incomplete, and biased ways to process information. Much more research has been done in this area and we would stand by the assertions, though perhaps stress positive aspects of using memorable events and recognise that it is difficult to take a comprehensive approach daily, even for statisticians.

Assertions 4–6 deal with people's handling of very low or very high probabilities. Micromorts and microlives are one way forward but this needs research.

Assertions 7 and 8 deal with equal likelihood and a tendency to transfer equal probability to unknown situations. These assertions are linked to the equiprobability bias of Lecoutre (1992) and remain areas for further exploration and should feature strongly in class discussion.

Assertions 9 and 10 involve incoherent assignment of probabilities, when probabilities are assigned to exclusive events but sum to more than unity. To know the tendency to supra-additivity can help students avoid that trap.

Batanero and Borovcnik (2016) identify archetypical forms of thinking behind the strong resistance of learners to improve their probabilistic strategies. In conclusion, in the time after covid, we repeat our plea above that multiple approaches including APT, FQT and SJT are essential in the teaching of risk and probability. We would suggest there is a stronger focus in education on examples of probability in everyday life. Martignon and Krauss (2009) is only one of research projects that show how probability, risk, and decisions may successfully be introduced for children as young as ten years. A continuation of the project of Chernoff and Sriraman (2014) on probabilistic thinking with more coverage of direct classroom research in the sense of our ten assertions from above is urgently due. Devlin (2014, p. xiii) states that

> "almost every actual application of probability is in the context of a one-off event, introducing the concept through atypical classroom experiments probably does more harm than good."

That gives a further orientation for future didactical research on probability: to focus more on decision-oriented approaches and to take the plurality of perspectives on probability of Chernoff and Sriraman (2014) serious and integrate subjectivistic (SJT) aspects of probability into a currently heavily frequentistically (FQT) overladen curriculum on probability.

References:

Batanero, M. & Borovcnik, M. (2016). *Statistics and probability in high school*. Rotterdam: Sense Publishers.

Blastland M, & Spiegelhalter, D. J. (2013). *The norm chronicles*. London: Profile Books.

Borovcnik, M., Kapadia, R. (2014). A historical and philosophical perspective on probability. In E. J. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking: Presenting plural perspectives. Advances in Mathematics Education, Vol. 7* (pp. 7-34). Berlin: Springer.

Borovcnik, M., Kapadia, R. (2018). Reasoning with Risk: Teaching probability and risk as twin concepts. In C. Batanero & E. J. Chernoff (Eds.), *Teaching and learning stochastics. Advances in Probability Education Research, ICME-13 Monographs* (pp. 3–22). Springer International Publishing.

Chernoff, E. & Sriraman, B. (Eds.) (2014). *Probabilistic thinking: presenting plural perspectives. Advances in Mathematics Education, Vol. 7*. New York: Springer.

Devlin, K. (2014). The most common misconception about probability? In E. J. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking: presenting plural perspectives. Advances in Mathematics Education, Vol. 7* (pp. ix–xiii). Berlin: Springer.

Gigerenzer, G. (2002). Calculated risks: How to know when numbers deceive you. New York: Simon & Schuster.

Kapadia, R., & Borovcnik, M. (Eds.) (1991). *Chance encounters: Probability in education*. Dordrecht: Kluwer.

Martignon, L. & Krauss, S. (2009). Hands-on activities for fourth graders: A tool box for decision-making and reckoning with risk. *International Electronic Journal of Mathematics Education (IEJME), 4*(3), M. Borovcnik & R. Kapadia (Eds.), Special issue on "Research and Developments in Probability Education", 227–258.

ONS (2021). *Deaths involving covid 19*, KU, March 2019 to April 2021, published 21 May 2021. London: Office for National Statistics

Schools Council (1980). *Teaching Statistics 11–16: Statistics in your world* (a handbook and a series of booklets). Slough: Foulsham.

The interested reader may find the bibliographical details of the following references (and further relevant references) in Borovcnik and Kapadia (2014, 2018):

Austrian Ministry of Health (2012), Borovcnik, M. & Kapadia, R. (2011a),
Borovcnik, M. & Kapadia, R. (2011b), Çınlar, E. (2011), de Finetti, B. (1992/1937),
Dubben, H.-H. & Beck-Bornholdt, H.-P. (2010), Hacking, I. (1975), Hansson, S. O. (2007),
Kahneman, D. & Tversky, A. (1979), Laplace, P. S. de (1951/1812),
Kolmogorov, A. N. (1956/1933), Maistrov, L. E. (1974), Resnik, M. D. (1987),
von Mises, R. (1919), von Plato, J. (1994).