Control Charts and Capability Analysis for Statistical Process Control

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Shewhart initiated the basic work on quality control by making a distinction between chance or common causes and assignable or special causes of variation. By locating and eliminating the special causes of variation a process can be brought under statistical quality control. For this purpose, he developed graphical techniques, known as control charts, based on the types of available data in manufacturing processes. These charts are also referred to as Shewhart control charts in modern literature. In this article we discuss the basics of the Shewhart control charts, their statistical theories and applications with suitable illustrations. These techniques appear more appropriate for developing countries mainly because of the availability of natural resources and cheap labor and these could, in turn, help enhance their economies in short time easily. Besides, we have explained the capability analysis for variable real-life data. This paper is expected to be of particular interest to those who look for a high-quality performance in both the manufacturing and the service sectors.

Keywords: Three-sigma limits of control charts, variable control charts, attributes control charts, specification limits, capability indices

Introduction

Control chart is a statistical tool to detect whether a process is under statistical control or not. As the appropriateness of a control chart depends on the availability of the data set the types of available data sets are explained. Illustrations of all control charts are added. Further, the capability of the process control is explained with illustrations. Besides, the current scenarios of statistical process control (SPC) in developing countries are discussed because this could help build their economies quickly. The main advantages of these places are the availability of natural resources and cheap labor.

2. Control Charts for Statistical Process Control

Though this statistical tool was initially developed to check the current status of the process of a manufacturing sectors by Shewhart (1924) but with the passage of time this technique has become very popular for examining the process of service sectors too. A process is said to be under statistical control if all points (statistics) lie within upper and lower control limits. But if a point exceeds upper control limit (UCL) or falls below the lower control limit (LCL) the process is said to be out of statistical control. In this situation all points that go outside the control limits are removed from the given data set and fresh control limits with the remaining observations are computed. Again, the remaining points are plotted with the new control limits. The process is continued until all points lie within the control limits. This is referred to as the final control chart while the first control chart is known as the trial control chart.

3. Types of Data and Control Charts

Usually we come across three types of data in both the manufacturing and service sectors. These include (i) variable or measurable data, where $\bar{x}$, R, and sigma charts are used (ii) attribute data (fraction defective or fraction nonconforming) where p chart is employed and (iii) the number of count of defects or nonconformities (defects) where c chart is found useful. Some authors consider the attribute data consisting of both fraction nonconforming and nonconformities. See Montgomery (2009) for more details.

3a. Control Charts for Variable or Measurable Data

For process average we have two types of $\bar{x}$ charts, one using range, R and the other involving standard deviation, σ. For process variability we have two charts, which include R chart and sigma chart. Let x_{ij} denote the jth measurement of the ith sample, j = 1, 2, . . . ,n and i = 1,2, . . . , k. The mean of the ith sample is denoted by $\bar{x}_i$ and it is computed as shown below. $\bar{x}_i = \frac{\sum_1^n x_{ij}}{n}$. The range of the ith sample is represented by R_i and it is obtained by taking the difference between its highest and the lowest value of the measurements, R_i = Max (x_{ij}) - Min (x_{ij}). Suppose μ denotes the population mean and σ^2 represents the population variance. Then, var($\bar{x}$) = σ^2/n and SE($\bar{x}$) $= \sigma/\sqrt{n}$, where SE stands for Standard Error.

3b. Three-Sigma Control Limits for R and $\bar{x}$ Control Charts

UCL= E($\bar{x}$) + 3 SE($\bar{x}$) = μ + 3 $\sigma/\sqrt{n}$, Central Line (CL) = E($\bar{x}$) = μ, LCL= E($\bar{x}$) - 3 SE($\bar{x}$) = μ - 3 $\sigma/\sqrt{n}$. Note that μ and σ are parameters (standards), which are usually not known. As such these are, in turn, replaced by their respective estimators. μ is estimated by $\bar{\bar{x}}$,which is the mean of the k means. It is computed by $\bar{\bar{x}} = (\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_k)/k$, where $\bar{x}_i$ is the mean of the ith sample. σ is estimated by $\bar{R}/d_2$ where $\bar{R} = (R_1 + R_2 + \dots + R_k)/k$ and d_2 is a constant that depends on the sample size n With these estimators, we obtain the control limits as UCL = $\bar{\bar{x}} + 3\bar{R}/(d_2\sqrt{n}) = \bar{\bar{x}} + A_2\bar{R}$, CL = $\bar{\bar{x}}$, LCL = $\bar{\bar{x}} - 3\bar{R}/(d_2\sqrt{n}) = \bar{\bar{x}} - A_2\bar{R}$, where $A_2 = 3/(d_2\sqrt{n})$. For detecting the lack in the process variability, usually R and sigma charts are used. Here we discuss R chart. For obtaining the control limits of this chart we have UCL= $D_2\sigma$, CL= $d_2\sigma$, LCL= $D_1\sigma$. As σ is also estimated by $\bar{R}/d_2$ we get UCL = $D_4\bar{R}$, CL = $\bar{R}$, LCL = $D_3\bar{R}$.The values of constant are available for various values of n in almost all standard text books of statistics.

Illustrations of R and $\bar{x}$ Control Charts

To study the aspect of service quality that relates to the amount of time it takes to deliver luggage (as measured from the time the guest completes check-in proceeds to the time the luggage arrives in the guest's room), data were recorded over a 4-week period. Subgroups of five deliveries were selected from the evening shift on each day for analysis. The data set given below summarizes the result for all 28 days. The hotel management has instituted a policy that 99% of all luggage deliveries must be completed in 14 minutes or less.

Day	Luggage Delivery Times in Minutes	Day	Luggage Delivery Times in Minutes

1	6.7	11.7	9.7	7.5	7.8	15	7.8	9.0	12.2	9.1	11.7
2	7.6	11.4	9.0	8.4	9.2	16	11.1	9.9	8.8	5.5	9.5
3	9.5	8.9	9.9	8.7	10.7	17	9.2	9.7	12.3	8.1	8.5
4	9.8	13.2	6.9	9.3	9.4	18	9.0	8.1	10.2	9.7	8.4
5	11.0	9.9	11.3	11.6	8.5	19	9.9	10.1	8.9	9.6	7.1
6	8.3	8.4	9.7	9.8	7.1	20	10.7	9.8	10.2	8.0	10.2
7	9.4	9.3	8.2	7.1	6.1	21	9.0	10.0	9.6	10.6	9.0
8	11.2	9.8	10.5	9.0	9.7	22	10.7	9.8	9.4	7.0	8.9
9	10.0	10.7	9.0	8.2	11.0	23	10.2	10.5	9.5	12.2	9.1
10	8.6	5.8	8.7	9.5	11.4	24	10.0	11.1	9.5	8.8	9.9
11	10.7	8.6	9.1	10.9	8.6	25	9.6	8.8	11.4	12.2	9.3
12	10.8	8.3	10.6	10.3	10.0	26	8.2	7.9	8.4	9.5	9.2
13	9.5	10.5	7.0	8.6	10.1	27	7.1	11.1	10.8	11.0	10.2
14	12.9	8.9	8.1	9.0	7.6	28	11.1	6.6	12.0	11.5	9.7

[Source: Levine et al. (2016). The data set was collected under *Analyze* phase of the Measure phase of Six Sigma project of Beachcomber Hotel.]

We need (i) to draw charts to examine the current status of the hotel service quality.(ii) to examine whether the process is capable of meeting the 99% goal set forth by the hotel management.(iii) to calculate the CPU for measuring the process performance.

Solution: (i) First we compute mean and range for each day, and these values are given in Table 1.

Table1. Mean and range of the delivery of luggage times for all 28 days

Day	Mean	Range	Day	Mean	Range
1	8.68	5.0	15	9.96	4.4
2	9.12	3.8	16	8.96	5.6
3	9.54	2.0	17	9.56	4.2
4	9.72	6.3	18	9.08	2.1
5	10.46	3.1	19	9.12	3.0
6	8.66	2.7	20	9.78	2.7
7	8.02	3.3	21	9.64	1.6
8	10.04	2.2	22	9.16	3.7
9	9.78	2.8	23	10.30	3.1
10	8.80	5.6	24	9.86	2.3
11	9.58	2.3	25	10.26	3.4
12	10.00	2.5	26	8.64	1.6
13	9.14	3.5	27	10.04	4.0
14	9.30	5.3	28	10.18	5.4

The R Chart

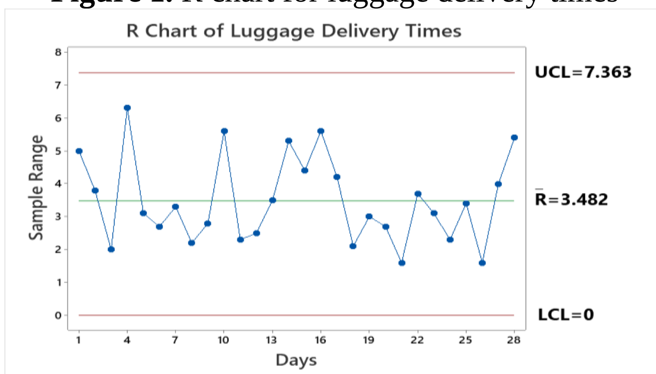
We obtain $\overline{R} = (R_1 + R_2 + \ldots + R_k)/k = 3.482$ The control limits for R chart are obtained as UCL $= D_4\overline{R} = 2.114*3.482 = 7.361$, CL $= 3.482$ and LCL$= D_3\overline{R} = 0$.R chart for luggage delivery times is depicted using Minitab 19 in **Figure 1**, which does not indicate any range value outside the control limits or any clear patterns. This means that the process is in statistical control, free from special cause of variation.

The $\bar{x}$ Chart

As the R chart shows that the process is in control with respect to the process variability we next plot the $\bar{x}$ chart to examine the current status of the process with respect to the process average.

For control limits of $\bar{x}$ Chart, we have $\bar{\bar{x}}$=265.38/28 = 9.478, A_2 = 0.577, $\bar{R}$ =3.482. Finally, we compute UCL= 9.478 + 0.577*3.482=11.487, CL= 9.478, LCL= 9.478 - 0.577*3.482 =7.469.
Figure 2 displays the $\bar{x}$ chart for the luggage delivery times, which does not disclose an average value of the luggage delivery time outside the control limits. This, in turn, helps conclude that the process is in statistical control with respect to the process average. As both R chart and $\bar{x}$ chart are in control we conclude that the luggage delivery process is in a state of statistical control.
The rest part of the illustration, (ii) and (iii) are explained under the illustration of Capability Analysis of Section 4.

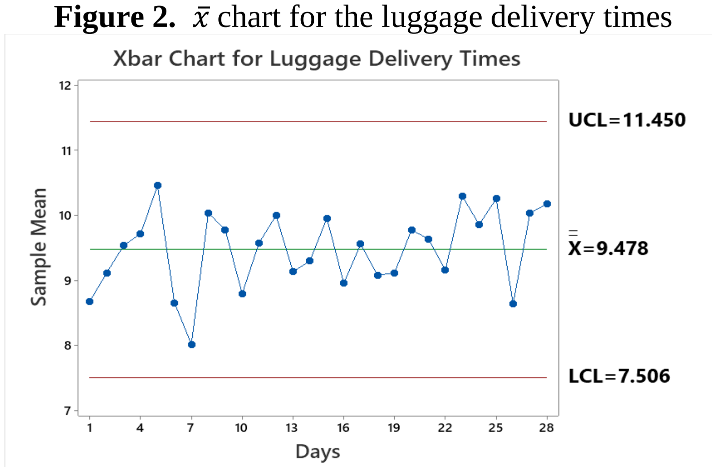
Figure 1. R chart for luggage delivery times



3c. Control Charts for Attributes

In manufacturing process attributes of products are referred to as fraction defective or proportion nonconforming and control charts are called as p-charts. The characteristics are observed by classifying the products into two classes or groups, usually said as defective or nondetective. Suppose P denotes the probability of an item being defective while (1-P) shows the probability of an item being nondetective.

Figure 2. $\bar{x}$ chart for the luggage delivery times



Let us suppose that the proportion of defective items in then sample of size n is given by p = $\frac{d}{n}$. It means that d defective items were found out of n items inspected. Then we have E($\frac{d}{n}$) = P and var ($\frac{d}{n}$)= $\frac{P(1-P)}{n}$. Thus, the control limits are given as UCL= P + 3 $\sqrt{\frac{P(1-P)}{n}}$, CL=P, LCL= P - 3

$\sqrt{\frac{P(1-P)}{n}}$. As P is usually unknown it is estimated by $\bar{p}$ where $\bar{p} = (d_1 + d_2 + \ldots + d_k)/nk$ where d_i defective items were found in the ith sample. Thus, on replacing P by $\bar{p}$ we obtain the following control limits as UCL= $\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$, CL= $\bar{p}$, LCL= $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Illustration of p chart

For 32 days, 500 film canisters were sampled and inspected. The given data set lists the number of defective canisters (the nonconforming items) for each day (the subgroup). Construct p chart and examine the state of statistical control. {Source Levin at al.(2016).]

Day :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Items:	26	25	23	24	26	20	21	27	23	25	22	26	25	29	20	19
Day :	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Items:	23	19	18	27	28	24	26	23	27	28	24	22	20	25	27	19

Solution

Here n = 500, $\bar{p}$ = Total number of nonconforming items/Total items inspected =761/500*32=761/16000=0.0476. Here UCL = 0.0761, CL=0.0476, LCL=0.0190. See **Figure 3**. As all points lie between UCL and LCL we conclude that the process is in a state of statistical control.

3d. Control Charts for Nonconformities (Defects) or c Chart

The failure to meet a specification at a point results in a defect or nonconformity. An item or a product having at least one defect or nonconformity is referred to as defective or nonconforming item. C chart is plotted for defects or nonconformities per unit,. The unit considered may be a single item or a group of items, part of an item, etc. There are numerous opportunities for defects to occur on an item but the probability of the occurrence of a defect is very small, For these reasons the statistical theory for c chart is based on the Poisson distribution. As mean = variance in this distribution, mean = c and variance = c. Thus, the control limits for nonconformities or c chart with standard given are given as UCL = c + 3 $\sqrt{c}$, CL= c, LCL= c - 3 $\sqrt{c}$.

As c being the parameter of the Poisson distribution is not known, it is estimated by the average value of the observed values of the nonconformities. Let us represent it by $\bar{c}$. The control limits of c chart with the estimated value as $\bar{c}$ we have UCL = $\bar{c}$ + 3 $\sqrt{\bar{c}}$, CL= $\bar{c}$, LCL = $\bar{c}$ - 3 $\sqrt{\bar{c}}$.

Figure 3. p chart of number nonconforming

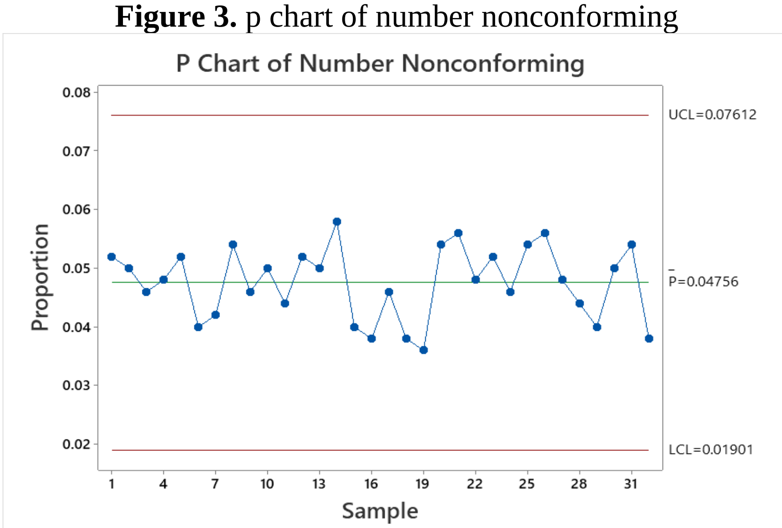


Illustration of c Chart

The following data were collected on the number of nonconformities per unit for 10 time periods.

Time:	1	2	3	4	5	6	7	8	9	10
Nonconformities per unit:	25	11	10	11	6	15	12	10	9	6

Construct the appropriate control chart and determine UCL, CL and LCL. What is the state of statistical quality control?

[Source: Levine et al. (2016)]

Solution: We get $\bar{c} = 11.5$ and UCL=21.6735, CL = 11.5, LCL= 1.3265

The control chart is shown in **Figure 4**. Yes, time 1 observation is above UCL that reveals that the process is not in statistical control because of the presence of some special causes of variation.

4. Capability Analysis

For analyzing the capability process, we estimate the percentage of products or services that are within specifications. Assuming that the process is in control and X is approximately normal, we can estimate the probability of a process outcome being within specifications.

If lower specification limit and upper specification limit are represented by LSL and USL respectively, then Prob (outcome within specifications) = Prob (LSL< X < USL)

$$= \text{Prob}\left[(LSL-\bar{\bar{x}})/\bar{R}/d_2 < Z < (USL-\bar{\bar{x}})/\bar{R}/d_2\right] \text{ where } Z=(X-\bar{\bar{x}})/\bar{R}/d_2$$

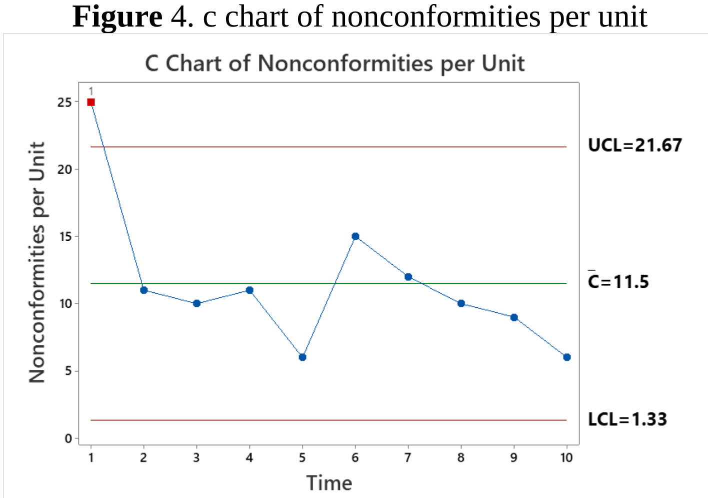
Illustration of Capability Analysis

See (ii) and (iii) part of the illustration of $\bar{x}$ and R charts.It is further mentioned that the hotel management has instituted a policy that 99% of the luggage deliveries must be completed in 14 minutes or less. Thus USL=14, n = 5, $\bar{\bar{x}} = 9.478$, $\bar{R} = 3.482$ and $d_2 = 2.326$.

Prob (outcome within only upper specification) = Prob $[Z < (USL-\bar{\bar{x}})/\bar{R}/d_2]$=Prob[Z<(14-9.478)/(3.482/2.326)] = Prob[Z<3.02] = 0.99874.

This suggests that 99.874% of the luggage deliveries will be made within the specified time. We, therefore, conclude that the process is capable of meeting the goal of 99% set forth by the hotel management. (iii) As CPU = $(USL - \bar{\bar{x}}) / 3(\bar{R}/d_2)$, we get CPU = 1.01. This indicates that the USL is slightly more than 3 standard deviations above mean.

Figure 4. c chart of nonconformities per unit



5. Scenarios in Developing Countries

SPC techniques played a very important role to help rebuild the economic conditions of Japan after the second world war under the supervision of W. E. Deming, a US expert of statistical quality control and improvement. In developing countries SPC techniques can help improve their economic conditions tremendously if employed properly because of their cheap labor and natural resources. Madanhire and Mbohwa (2016) described the applications of SPC in Zimbabwean manufacturing industries without mentioning a data set. They also discuss the existing associated problems with manufacturing industries there, which could be minimized. Abtew et al. (2018) used the data set of garments producing company known as 'Silver Spark Apparel Limited (SSAL)', a Raymond Group Company, India for an illustration. But the used data set is not given in their paper. There are good garment companies in Ethopia, China, India, etc. and other industries in developing countries, which could contribute to the incomes of their countries considerably using the SPC techniques. These countries could follow the example of Japan for achieving their goals

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