Modeling interference with α-stable distributions and copulæ for receiver design in Wireless Communications

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Abstract

Interference (signals coming from undesired transmitters) is a limiting factor in many wireless communication situations and especially for Internet of Things. It is often considered as a Gaussian random variable with independent components but these assumptions (Gaussian and independent) are no longer available in many cases. A better model of interference can be the α-stable distribution with copulæ to take the dependence structure into account. The main aim of this paper is to obtain an evaluation (explicit or numeric) of the Log-Likelihood Ratio (LLR) in this interference model. We show that the copula framework allows us to split the LLR into a part depending only on the copula and the marginal distribution of the interference and another which is exactly the LLR for independent noise (that only depends on the marginal distributions). To have closed-form solutions, we will focus on the case where the noise follows a Cauchy distribution, which is an α-stable distribution with $\alpha = 1$. We also consider the Archimedean copula family which also allows to have tractable solutions. We can then observe the impact of impusive noise and dependence on the decision regions.

Keywords: α-stable distributions; Copulæ; Log-Likelihood Ratio; Wireless Communication.

1. Introduction

Impulsive interference is encountered in many situations, e.g. in power line communications, with ultra-wide band technology, or in dense networks. In the presence of interference, classical receivers are not robust enough and the quality of the transmission is degraded (BenMâad 2010).

In this paper, we consider a general detection problem in a block fading scenario. Each data symbol is transmitted over wireless channels and K versions of each symbol are received. This transmission structure can be motivated by many different practical wireless communication systems, for example a rake receiver or a single-input-multiple-output system for instance. For a transmitted symbol s_k at time k, the received signal $\mathbf{y}_k \in \mathbb{R}^K$ is:

$$\mathbf{y}_k = s_k \mathbf{h}_k + \mathbf{I}_k + \mathbb{N}_k, \tag{1}$$

where $\mathbf{h}_k \in \mathbb{R}^K$ is the block fading channel coefficients, $\mathbf{I}_k \in \mathbb{R}^K$ is the impulsive interference and $N_k \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$ is the thermal noise.
In this paper we assume that interference is dominating. Besides we assume independence between different time instants k so that we will drop this index for simplicity of writing. The studied case can then be summarized by

$$\mathbf{Y} = s\mathbf{h} + \mathbf{I} \tag{2}$$

where s is the transmitted symbol, $\mathbf{h}$ is a vector containing the attenuations of the different channel carrying the information and $\mathbf{I}$ the interference vector.

Many papers have considered the case where $\mathbf{I}$ is composed of independent and identically distributed samples. Depending on the impulsive interference distribution assumption, it is more or less complicated to derive the optimal receiver and sometimes suboptimal approaches are considered, especially when $\mathbf{I}$ is considered as an α-stable random vector.
The main goal of this paper is to propose a new modelling approach for the interference, including its non Gaussian nature and the dependence structure. With the proposed model we can derive an evaluation (explicit or numeric) of the Log-Likelihood Ratio (LLR). We show that the copula framework allows us to split the LLR into a part depending only on the copula and the marginal distribution of the interference and another which is exactly the LLR for independent noise (that only depends on the marginal distributions). To have closed-form solutions, we will focus on the case where the noise follows a Cauchy distribution, which is an α-stable distribution with $\alpha = 1$. We also consider the Archimedean copula family which also allows to have tractable solutions. We can then observe the impact of the dependence on the decision regions.

2. System and models

2.1. System model

The received signal $\mathbf{Y} \in \mathbb{R}^K$ is given in (1). We consider a binary source $s \in \{-1, 1\}$. The block fading channel coefficients are a random vector (RV) denoted by $\mathbf{h} \in \mathbb{R}^K$. The distribution of the coefficients depends on the considered channel model (e.g. Rayleigh, Nakagami, Rician etc.). We assume perfect channel state information at the receiver. The impulsive interference is denoted by a RV $\mathbf{I} \in \mathbb{R}^K$.
With these assumptions, the Maximum Likelihood (ML) detector is given by:

$$\Lambda = \log \frac{\mathbb{P}_{\mathbf{Y}}(\mathbf{y}|s = 1, \mathbf{h})}{\mathbb{P}_{\mathbf{Y}}(\mathbf{y}|s = -1, \mathbf{h})} \underset{\hat{s}=-1}{\overset{\hat{s}=1}{\gtrless}} 0. \tag{3}$$

2.2. Interference model

In many previous papers, it has been shown that, in several contexts, the interference term is not adequately modelled with a simple Gaussian distribution assumption. Middleton (1977, 1999) obtained general expressions based on series expansions and classified in three main categories depending if the noise bandwidth is less than the useful signal (class A) or greater (class B). class C is a sum of class A and B. However, Middleton models have been widely used in different contexts (MIMO (Chopra 2009), OFDM (Ishikawa 2007) or power line communications (Andreadou 2010)).
This popular model is challenging to work with since the density function is a doubly-infinite and alternating series of terms that are not easily computable. In practice, several approximation models have been proposed. The main approach is to consider only the most significant terms. For instance, it is claimed in Vastola 1984 that, in many situations for the class A, two or three terms can be sufficient to obtain a good approximation leading to a Gaussian mixture (Guney 2006). The two terms case is often denoted as the ϵ-contaminated noise (Aysal 2007) or Bernoulli-Gaussian noise (Herath 2012, Vu 2014).
More recently, many works have been done concerning Time Hopping Ultra Wide Band (TH-UWB). Empirical choices that allow analytical analysis of the receiver, justified by simulations, observations of the estimated PDF and/or gains in BER have been proposed: Gaussian and Laplace mixtures (Beaulieu 2010), Generalized Gaussians (Fiorina 2006) or Gaussian mixtures (Hu 2008). Some surveys can be found in (Beaulieu 2009).
Another class of model of direct relevance to interference modelling is the α-stable. It has often been used in the UWB context (Win 2006, Beaulieu 2009, ElGhannudi 2010). But on the contrary to the previously

discussed approaches, it relies (when no power control is done) on a theoretical derivation (that can be related to a physical interpretation), closely linked to Middleton's work and finding its foundation in stochastic geometry (Weber 2012). In an *ad hoc* network, an unbounded received power[1] assumption makes the interference fall in the attraction domain of a stable law.

The proof of this result is generally done considering the log-CF of the total interference, see for instance Sousa 1992, Win 2009 or ElGhannudi 2010, which can be written as: $\psi_I(\omega) = \log\left(\mathbb{E}\left[\exp\{j\omega^T I\}\right]\right) = -\delta^\alpha |\omega|^\alpha$, where I is the total interference and T denotes the transpose. The right term is the log-characteristic function of a symmetric α-stable (SαS) random variable with dispersion δ. Another solution for the proof, based on the Lepage series, was proposed in Ilow 1998.

2.3. Dependence structure

Copulæ are a very useful way to model structures of dependence between random variables (Nelsen Z007). The fundamental result justifying this usefulness is the Sklar's Theorem: it ensures that under the condition that the cumulative distributions of the random variables are continuous, there exists a unique copula C such that $\forall(x_1, \ldots, x_d)$, we have

$$H(x_1, \ldots, x_d) = C\left(F_1(x_1), \ldots, F_d(x_d)\right). \tag{4}$$

where H is the joint distribution of $(X_1, \ldots, X_d)$. Hence, a copula is a function $C : [0,1]^d \mapsto [0,1]$ which couples the marginals F_i between themselves.

Archimedean copulæ: we consider the particular class of bivariate Archimedean copulæ. The interest of this class is, first of all, the easiness with which they can be constructed. The multivariate Archimedean copulæ have the following form: for all $(u_1, \cdots, u_d) \in [0,1]^d$,

$$C(u_1, \cdots, u_d) = \phi^{-1}\left(\phi(u_1) + \cdots + \phi(u_d)\right). \tag{5}$$

The function ϕ is called the generator of the copula and is a continuous and convex function such that $\phi(1) = 0$. It appears that all Archimedean copula is symmetric in its variables. We focus on two families of Archimedean copulæ, both indexed by a single parameter. The Clayton (ϕ^{-1} is the Laplace transform of a Gamma distribution) and the Gumbel families (ϕ^{-1} is the Laplace transform of a α-stable distribution) of copulæ model asymmetric dependence in tails.

Clayton: For all $\theta > 0$, the Clayton copula of parameter θ is defined on $[0,1]^d$ by

$$C(u_1, \cdots, u_d) = \left(u_1^{-1/\theta} + \cdots + u_d^{-1/\theta} - (d-1)\right)^{-\theta}.$$

Gumbel: For all $\theta \geq 1$,The Gumbel copula of parameter θ is defined on $[0,1]^d$ by

$$C(u_1, \cdots, u_d) = \exp\left(-(\sum_{i=1}^{d}(-\log u_i))^{1/\theta}\right).$$

3. LLR for dependent variables

In the two-dimensional case and with a binary input and a perfect channel state information (so that the $\mathbf{h}$ is simplified), our system model can be written

$$\begin{cases} y_1 = s + i_1 \\ y_2 = s + i_2 \end{cases}, \tag{6}$$

[1]the received power law is proportional to $d^{-\sigma}$, where σ is the channel attenuation coefficient and d the transmitter receiver distance; it tends to infinity when d tends towards zero.

where $s \in \{-1, 1\}$. Two repetitions y_1 and y_2 of this transmitted bit are obtained and $\mathbf{I} = (i_1, i_2)$ is a bivariate interference vector. The two coordinates i_1 and i_2 are not independent. Let f be the joint density of the couple (i_1, i_2). The Log Likelihoood Ratio (LLR) for each $\mathbf{Y} \in \mathbb{R}^2$ is given by

$$\Lambda(y_1, y_2) = \log \frac{\mathbb{P}(y_1 = s + i_1,\, y_2 = s + i_2 \mid s = 1)}{\mathbb{P}(y_1 = s + i_1,\, y_2 = s + i_2 \mid s = -1)} = \log \frac{f(y_1 - 1, y_2 - 1)}{f(y_1 + 1, y_2 + 1)}. \tag{7}$$

Independent interferences: in the left plot in Fig. 1, we illustrate the two decision regions in the case when interference is independent on the two dimensions and Cauchy distributed with location $x_0 = 0$ and scale $\delta = 1$. TA Gaussian noise would result in a linear boundary, corresponding to an Euclidean distance. Impulsiveness significantly modifies those boundaries and necessitate non linear operation to implement an optimal receiver.

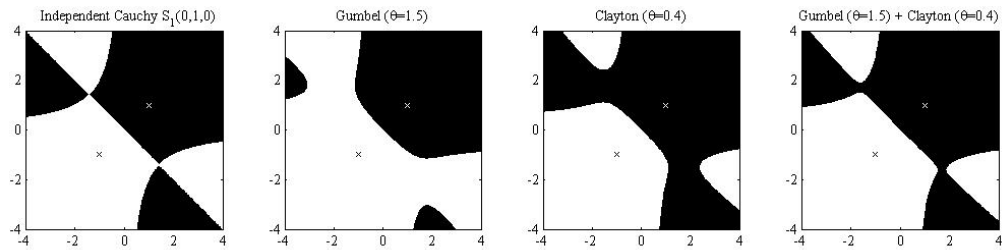


Figure 1: The X and Y axis are the values of the components of the received vector $\mathbf{Y}$. We consider two possible transmitted symbols $\{-1, 1\}$ meaning that the transmitted vector corresponds to the points $(1, 1)$ and $(-1, -1)$, denoted by two crosses on the figure. The white region corresponds to the decision 1, meaning that $\Lambda \geq 0$ and the black one to -1, i.e., $\Lambda < 0$ Decision region for independent Cauchy, Cauchy marginals and Gumble, Clayton or a mixture of Clayton and Gumbel copula.

Dependent interferences: if we now consider that i_1 and i_2 are dependent and that we can express this dependence through a copula, the LLR will become

$$\begin{aligned} \Lambda(x, y) &= \log \frac{f_i(x-1) f_i(y-1) c(F_i(x-1), F_i(y-1))}{f_i(x+1) f_i(y+1) c(F_i(x+1), F_i(y+1))} \\ &= \Lambda_{\perp}(x, y) + \Lambda_c(x, y), \end{aligned} \tag{8}$$

where c is the density of the copula and is defined by

$$c(u, v) = \frac{\partial^2 C}{\partial u \partial v}(u, v); \tag{9}$$

f_i and F_i are respectively the probability density function and the cumulative distribution of the interference marginals. $\Lambda_{\perp}$ represents the independent part of the LLR. The second term

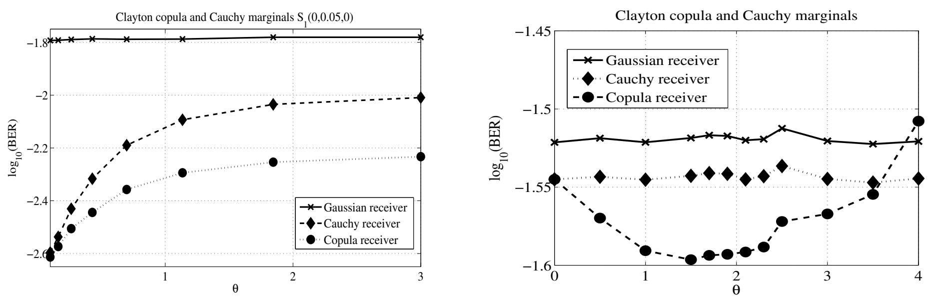
$$\Lambda_c(x, y) = \log \frac{c(F_i(x-1), F_i(y-1))}{c(F_i(x+1), F_i(y+1))} \tag{10}$$

is the part of the LLR depending on the copula and represents the dependence structure. It can however be tricky to derive because it also depends on the marginals.

The impact on the decision region is represented in Fig. 1. The Gumbel and Clayton cases clearly modify these regions. The significant impact is in part due to the asymmetry in the dependence, increasing the impact of, respectively, the upper or the lower tail. Because in wireless communication, interference is generally symmetric, we propose a mixture of Gumbel and Clayton to improve the performance. The impact on the decision region still exists but is less significant. This could be different in higher dimensions.

4. Receiver design

Perfectly known interference: the optimal receiver in terms of minimizing the Bit Error Rate (BER) is the Maximum Likelihood (ML) detector $\hat{s} = \text{argmax}_{s\in\Omega}\mathbb{P}(\mathbf{Y}|s)$, where Ω is the set of possible transmitted bits. In the binary case, $\Omega = \{-1, 1\}$, the problem is reduced to obtaining the sign of the LLR defined in (8): $\widehat{s} = \text{sign}\left(\Lambda(x, y)\right) = \text{sign}\left(\Lambda_{\perp}(x, y) + \Lambda_c(x, y)\right)$. Fig. 2a compares the performance of the linear Gaussian receiver, a Cauchy receiver assuming independent Cauchy interference and a copula receiver that knows both the marginal and the dependence structures. In that case the dependence is captured by a Clayton copula. Obviously when the parameter gets close to zero, the dependence is low, the Cauchy receiver outperforms the Gaussian receiver and there is no need to introduce the dependence structure. However, when the dependence increases (θ gets larger), the performance of the Cauchy receiver quickly degrades when the copula receiver is able to maintain a better performance level. A similar conclusion would be obtained with the Gumbel case.



(a) (b)

Figure 2: BER for (a) Cauchy noise and Clayton copula as a function of the Clayton parameter and (b) SIMO model and a receiver based on a mixture of Clayton and Gumbel copulæ.

Application to SIMO transmissions: We finally apply our Copula receiver to the SIMO case. Interferers are uniformly distributed in a square around the receiver. The square is 10m by 10m and the mean number of interferers is 50. The channels are Rice channels with a main path strength randomly chosen. The channel attenuation coefficient is 3. For the copula receiver, we chose a mixture of the Gumbel and Clayton copulas to ensure a symmetry in the upper and lower tail dependence. To have the symmetry, the parameters for the two copula are linked so that only one parameter has to be chosen to define the dependence structure. To observe the impact of including the dependence in the receiver, we vary this parameter. Fig. 2b shows the benefit of including the dependence structure into design the receiver.
The gain in performance is limited but this is easily explained by the small dimension considered (only 2). Besides, we chose a Cauchy distribution for the marginals which is not the optimal choice. It has however proved to be close to the optimal in several situations.It is clear that taken the dependence structure into account allow a further gain compared to the independent receiver, which already gives better performance than the Gaussian receiver.

Conclusion

We proposed in this paper a way to model interference with α-stable distributions and a copula for the dependence structure. In the case of Cauchy marginals and copulæ from the Archimedean family and with a binary input, we are able to derive analytical expressions of the decision rule based on the likelihood ratio and to esily implement the receivers. The results on the decision regions show that dependent interference has a significant impact on the optimal decision that we should make. Consequently, we compared receivers that takes this dependency into account to receivers that do not. We show that the latter can rapidly degrade if a dependence structure is present when the former manage to maintain good performances. We illustrate the possible benefit on a SIMO example.

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