

Some Bayesian Approaches to Model Robust Designs

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1. Abstract

In industrial experiments, it is natural to look at many factors simultaneously. Cost considerations will sometimes make it impractical to design experiments so that effects of all these factors are estimated simultaneously. Therefore experimental designs are frequently constructed to estimate main effects and a few pre-specified interactions. However, a criticism frequently associated with the use of optimality criteria such as D or G-optimality is the specific reliance on an assumed statistical model f . One way to deal with such a criticism may be to assume that instead the true model is contained in $F = \{f_1, f_2, \dots, f_d\}$. Here we consider a class of designs that can be used for estimation of main effects and any combination of up to g interactions where g is specified by the user. We view this as an issue of model robust design. Methods have been proposed for constructing and analyzing such designs using frequentist approaches.

This thesis is motivated by the belief that appropriate Bayesian approaches may perform well in constructing model robust designs and by the lack of such approaches in the literature. I introduced two ideas: the first one uses the traditional Bayesian design method for parameters estimation and incorporates a discrete prior probability on the set of models of interest. The second one extends the applicability of the model discrimination design method developed by De Leon and Atkinson (1991) to more than two models. The latter is more adapted to non linear and response surface designs with one predictor. Some examples and comparisons with existing approaches will be provided.

REFERENCES

A.C. Atkinson (1972), Planning experiments for discrimination between models. *J. Roy. Statist. Soc. Ser. 36*, 321-334.

A.C. Atkinson and A. N. Donev (1992), *Optimum Experimental Designs*. Oxford Statistical Science Series: 8, 1-328.

G. E. P. Box and R. D. Meyer (1986), An Analysis for Unreplicated Fractional Factorials. *Technometrics* 28, 11-18.

R.J. Brooks (1972), A decision theory approach to optimal regression designs. *Biometrika* 59, 563-571.

K. Chaloner (1984), Optimal Bayesian experimental designs for linear models. *Ann. Statist.* 12, 283-300.

K. Chaloner (1985), Bayesian experimental design: A review. *Statistical Science*

10, 273-304. K. Chaloner and K. Larntz (1986), Optimal Bayesian designs applied to logistic regression experiments. *J. Statist. Plann. Inference* 21, 191-208.

Chipman, M. Hamada and C. F. J. Wu (1997), A Bayesian Variable Selection Approach for Analyzing Designed Experiments with Complex Aliasing. *Technometrics* 39, 372-381.

M.A. Clyde and K. Chaloner (1996), The equivalence of constrained and weighted designs in multiple objective design problems. *J. Amer. Statist. Assoc.*

R. D. Cook and C. Nachtsheim (1980), A comparison of algorithm for constructing exact D-optimal designs. *Technometrics* 22, 315-324.

R. D. Cook and C. Nachtsheim (1982), Model Robust, Linear-Optimal Designs. *Technometrics* 24, 49-54.

A. DasGupta and W. J. Studden (1994), Robust Bayes designs in normal linear models *Ann. Statist* 9, 1244-1256.

RÉSUMÉ

Dans les experimentations industrielles, il est naturel de s'interesser a plusieurs factors simultanement. Les considerations de couts font que parfois il est difficile de plannifier des experimentations de maniere a estimer tous les facteurs simultanement.

Alors, les experimentations sont frequemment plannifiees de de maniere a estimer les effects primaires et un nombre determine d'interactions. Neanmoins, une critique associee avec l'utilisation des criteres d'optimalite telles que le D ou le G-optimalite est leur appui sur une modele statistique f . Une maniere de faire face a cette critique est de considerer qu'en fait la vraie modele appartient a un ensemble $F = \{f_1, f_2, \dots, f_d\}$. Nous considerons une class de plannifications qui peuvent etre utilisees pour l'estimation des principaux effects et un nombre specifique g d'interactions ou g est specifie par l'experimentateur. Nous considerons cette situation comme etant celle des modeles de plannifications robustes. Des methodes ont ete pour construire de telles plannifications et nous proposons ici une approche Bayesienne.

J'introduirai deux idees: la premiere sera d'utiliser l'approche Bayesienne traditionnelle pour l'estimation des parametres en tenant compte du fait qu'il y a plusieurs modeles et en associant des probabilites a chacunes des models en question; la deuxieme sera d'etendre la methode de discrimination des modeles a plus de deux modeles.