

Agrometeorological Crop Yield Forecasting Methods

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ABSTRACT: This paper presents an introduction to agrometeorological crop yield forecasting (ACYF) methods. Such methods attempt to quantitatively assess the effect of weather vagaries on regional crop yields before harvest. The paper insists on the fact that all ACYF approaches are eventually calibrated against traditional yield statistics obtained through sampling procedures.

Three methods are covered: descriptive methods, regression methods and crop simulation methods. Descriptive methods simply classify weather conditions according to one or several variables to identify conventional thresholds separating groups of significantly different yields. Regression methods derive equations relating crop yield with weather variables. Finally, simulation methods, the most complex ones currently in use, attempt to analytically describe the physical and physiological impact of environment and management conditions on crop development, growth and yield.

A large section of the paper deals with methodological options and questions. Most of them are linked with the differences in temporal and spatial scales of the model inputs and outputs, and apply equally to the three methods. Inputs cover a large spectrum of data sources and types, from weather stations (points) and satellites (pixels) to soils (polygons) and direct crop observations at the field level. The integration of the data requires techniques of aggregation and dis-aggregation which are now available, although their implementation is fraught with methodological problems. In practice, methods will be selected based on their performance under a range of different conditions and their cost of implementation.

1. Introduction

Agrometeorological crop yield forecasting (ACYF) methods provide a quantitative estimate of the expected crop yield over a given area, in advance of the harvest and in a way that constitutes an improvement over trends, provided no extreme¹ conditions occur. They are based on the common sense assumption that weather conditions² are the main factor behind the inter-annual (short-term) variations of detrended crop yield series [Petr 1991].

The wording “common sense assumption” is used because it is often difficult to show the direct relation between weather and crop yields at the higher levels of aggregations at which agricultural statistics are published (national level, states, regions, provinces, etc.), *unless* the area covered is very limited (small island countries) or a dominant limiting factor actually controls yields, for instance water availability in many semi-arid countries (like the west-African Sahel). Even the weather-dependent agriculture of countries like Bangladesh does not appear to be macroscopically weather dependent in published statistics, because of the aggregation of the three cropping seasons that cover one calendar year, and because of very different crop (mainly rice) typologies (irrigated, rain-fed, floating, etc.)

The variations affecting detrended yield series are often termed “random” by agricultural statisticians. Agrometeorological crop yield forecasting methods can reduce the random component by taking advantage of both (i) the knowledge of the direct and indirect mechanisms governing plant-weather interactions and (ii) the fact that, although they are largely unpredictable beyond a week or so, weather fluctuations are not random (they follow known patterns).

¹ In this context, “extreme” stands for conditions which are statistically infrequent, which involve larger than usual energies, such as in tropical cyclones. WMO includes pest outbreaks (desert locust) among extreme agrometeorological events.

² Weather refers to the condition of the atmosphere as actually observed at a given moment, while Climate describes “normal” conditions, i.e. average weather conditions.

This paper focuses on area-wide yield (also known as regional yield), i.e. the very values estimated by agricultural statistics through sampling techniques. Some ACYF methods have the potential to be applied at the farm and field levels, where they constitute powerful management tools. The application of ACYF at the micro-level will not be discussed.

Finally, it should be underlined that the yardstick against which all ACYF methods are measured and calibrated are the traditional agricultural statistics; strictly speaking, the agrometeorological methods thus forecast agricultural statistics. An immediate consequence is that agrometeorological forecasts reproduce all the errors and biases that may affect the agricultural statistics, plus some others related to the agrometeorological methods themselves.

The UN Food and Agriculture Organization (FAO) has been involved in agrometeorological crop monitoring and forecasting since the mid-seventies, when a series of droughts seriously affected crop production in west Africa. For a general overview of the FAO approach, the reader is referred to Frère and Popov [1986], Gommès [1997], and Gommès, Snijders and Rijks [1998].

2. Different Approaches to Agrometeorological Yield Forecasting

Agrometeorological crop forecasting methods can be classified according to many criteria, for instance the type of inputs they require, the scale at which they apply, their capacity to perform correctly under unusual environmental conditions, their cost of development and of implementation, the level of analytical and statistical sophistication, etc. There is a continuum between “agrometeorological” methods and other techniques, and many border cases can be identified³.

This paper schematically distinguishes three levels of sophistication, from *descriptive methods* to *regression methods* and to *crop simulation methods* (analytical modeling). Each of the methods can yield excellent results according to crop, location, etc., and the more complex models typically include elements or techniques from the lower levels. For instance, a crop simulation method could include sub-models that would fall under the descriptive methods, and will normally be calibrated against agricultural statistics using traditional regression techniques, as indicated in the introduction.

2.1 Descriptive Methods

The simplest descriptive methods are those that involve one or two thresholds. A hypothetical example is given in Table 1.

Table 1: Hypothetical example of wheat yield (tons/ha) dependence on two climatological variables, with 95% confidence interval

March total rainfall	June average sunshine hours per day	
	6 hours and less	more than 6 hours
75 mm and less	5 ± 1	6 ± 2
More than 75 mm	8 ± 1	10 ± 2

Descriptive methods are non-parametric. It is sufficient to identify the environmental (agrometeorological) variables that are relevant for the crop under consideration. This is normally

³ A typical example would be the atmospheric pollen count method described by Besselat and Cour [1997].

done with statistical clustering analysis on a combination of time-series and cross-sectional data. Once the groups have been identified, it must be verified that yield averages corresponding to different clusters significantly differ from each other.

One of the reasons why simple descriptive methods can be very powerful is that climate variables do not vary independently and constitute a “complex”⁴. For instance, low cloudiness is associated with high solar radiation, low rainfall, high maximum temperatures and low minimum temperatures. Each of the variables affects crops in a specific way, but since they are correlated, there is also a typical combined effect, which the non-analytical descriptive methods can capture.

The descriptive methods have a number of advantages: (i) to start with, no assumption is made as to the type of functional relationship between the variables and the resulting yield; (ii) the clustering takes into account the fact that many climatological variables tend to be inter-correlated, which often creates methodological problems, at least with the regression methods described below; (iii) confidence intervals are easy to derive and, once developed, the descriptive methods require no data processing at all; their actual implementation is extremely straightforward.

Many “El Niño” impacts on agriculture, which are currently debated, could best be treated by descriptive methods. El Niño effects on agriculture result from a long series of effects (El Niño → Global atmospheric circulation → Local weather → Local crop yield) where each step introduces new uncertainties. As mentioned above, this chain of interactions can also be seen as a “complex” starting with the El Niño - Southern Oscillation (ENSO) index. In southern Africa, for instance, warm El Niño events are associated with an premature start of the rainy season followed by a drought at the time of flowering of maize, the main crop grown in the area. This pattern usually results in good vegetative growth followed by drought induced crop losses. Cane, Eshel and Buckland [1994] have found good relations between El Niño parameters (the very beginning of the causal chain) and maize yields in Zimbabwe, which constitutes a good illustration of the concepts described in this section on “descriptive” methods.

Descriptive methods have also been used successfully to estimate the quality of agricultural products such as wine. Given that the concept of “quality” is difficult to describe in quantitative terms⁵, the non-parametric approach is probably the most suitable.

2.2 Regression Methods

The simplest regression techniques rely on regression equations (mostly linear) between crop yield and one or more agrometeorological variables, for instance:

$$\text{Yield (tons/ha)} = 5 + 0.03 \times \text{March rainfall (mm)} - 0.10 \times \text{June temperature (}^{\circ}\text{C)}.$$

Beyond their simplicity, their main advantage is the fact that calculations can be done manually, and in the fact that data requirements are limited. The main disadvantages lie in the fact that they perform very poorly outside the range of values for which they have been calibrated. They often also lead to unrealistic forecasts when care is not taken to give greater priority to the agronomic significance than to statistical significance. The equation above, for instance, suggests that low March rainfall (a negative factor) could be corrected by below zero temperatures in June (frost), which obviously does not make

⁴ This is not unrelated with the typical “weather types” described by meteorologists.

⁵ Quality of wine is described by a combination of pH, sugar contents and types, acid types, concentration of tannins, color, etc.

sense. Another disadvantage is connected with the need to derive a series of equations to be used in sequence as the cropping season develops. For an overview of regression methods, including their validation, refer to Palm and Dagnelie [1993] and to Palm [1997].

Many of the disadvantages of the regression methods can be avoided when value-added variables are used instead of the raw agrometeorological variables. Such a value-added variable would be, for instance, actual crop evapotranspiration, a variable known to be linked directly with the amount of solar radiation absorbed by the plant under satisfactory water supply conditions.

2.3 Crop Simulation Methods

Crop simulation methods are the most accurate and the most versatile in that they attempt to describe the crops behavior (physiology, development) as a function of environmental conditions. They thus tend to be less sensitive to “new” situations, i.e. situations that did not occur during the period used to “train” the model. For a comprehensive inventory and review of existing models, refer to Plentinger and Penning de Vries [1995]. For a detailed example of application to regional forecasting, see Dallemand and Vossen [1996].

Typically, simulation models incorporate several components or sub-models [Jones, Thornton and Hill 1997]:

- Phenology (or development), the qualitative differentiation of plant organs into stem, leaves, flowers, etc.;
- Assimilation, the rate at which organic matter is synthesized;
- Partitioning and respiration, the way the organic matter synthesized above is distributed between the plant organs;
- Root growth, the way roots explore a volume of soil from where water and nutrients are absorbed;
- Water management, essentially the way water infiltrates the soil, is absorbed by roots and is returned to the atmosphere either by transpiration through the crop or by evaporation from the soil; and
- Nutrient management, the absorption of nutrients through roots and their distribution between plant organs.

All the components above are interrelated and described by quantitative variables (*internal model variables*) and *model constants* such as dry plant matter stored in the roots and other organs, total leaf area and growth rate, respiration rate, etc. Every variable is re-computed at regular time steps, from minutes to 10-day periods, which vary according to the level of sophistication of the models. All variables are clearly interdependent. For instance, leaf development depends on the rate of assimilation, but also conditions the rate at which water is transpired, which in turn affects leaf growth. The model variables are crop- and variety-specific in most cases.

Next to model variables and constants, models are also characterized by the inputs they require, essentially environmental conditions which change from year to year and which actually drive the model. They include weather factors (rainfall and radiation or sunshine play the main part), soil data (fertility, water holding capacity, etc.) and the crop data which are under the control of farmers such as planting dates, rate of application of fertilizer, etc. It is now also customary to replace one or more of

the sub-models by more or less directly observed variables. Examples include crop phenology and assimilation, based vegetation or greenness indices derived from earth-resources satellites, or soil moisture, which constitutes one of the essential components of the “water management” component.

A model run simulates the outcome (yield) of the plant-environment-management system for one set of model variables, environmental variables and management inputs. Strictly speaking, this includes only a limited number of situations which could apply only to one very specific field. As the objective of ACYF is to estimate yield at the regional level, several techniques must be applied to convert the yield output by a model to regional averages. This is described under 4 below.

3. Using Models as a Forecasting Tools

Crop simulation models provide an estimate of the likely yield as a function of the environmental and management conditions throughout the crop cycle (and before - soil moisture and soil fertility also depend on the conditions before the crop was sown). However, when forecasting crops, only the conditions up to the time of the forecast are actually known. It is thus necessary to provide the model with “future” data as well.

Two basic approaches can be adopted: (i) use of average (climatic normal) between the time of the forecast and harvest, and (ii) use of the data from the N latest years on record in turn, which results in N yield estimates, the average of which (and the resulting confidence interval) are then assumed to constitute the envelope of the current year’s estimates. Particularly when models are used for short-term management decisions, the longest available forecast (roughly up to 10 days) is often inserted between the past observed data and the “future” data. However, this involves some methodological and practical difficulties⁶ as well which are hardly justified for most regional yield forecasts.

Instead of historical data, it is also possible to use synthetic time series provided by Random Weather Generators (RWG), computer programmes which produce time series with the same statistical properties as the actual data. It is often believed that long RWG series can help reduce the error affecting crop forecasts⁷. Finally, RWG can also produce sets of coherent climatic parameters (rainfall, temperatures, wind, etc.) on a spatial grid (see point 4 below).

RWGs can also prove useful whenever the historical weather series are affected by a long-term trend, as in Sahelian rainfall from about 1960 to 1984 (when the trend reverted and became positive⁸). It is clear that the historical data used in a forecast must be as recent and short as possible, to account for short-term fluctuations, but still long enough to retain statistical significance. RWGs can be “trained” with the last ten years of data, then generate a much longer series (e.g. 50 years) to be used in a forecast.

Most crop forecasts, regardless of the technology adopted, take only into account the direct environmental factors. This means that the effect of large scale pest attacks, for instance, must be

⁶ Among others, the facts that weather forecasts do not normally include rainfall amounts nor provide useable details about the spatial distribution of the predicted weather.

⁷ For a more systematic treatment of errors affecting ACYF models, refer to Hough, Gommès and Keane [1998].

⁸ Even in developing countries, farmers react very quickly to climate fluctuations. By 1984, when the Sahelian drought reached its peak, groundnuts had completely disappeared from Mauritania. 1988 was the first “good” year, and groundnuts were back in 1989.

assessed separately. Of course, such attacks are also open to simulation but, as with extreme events, the statistical base (number of historical data available for model calibration and verification) is very weak.

Chronic pest attacks and unusual weather conditions of limited spatial extent and intensity are included in the calibration process.

4. Some Methodological Questions and Options⁹

The conversion of model output to regional yields always requires some kind of tuning, which involves an empirical statistical comparison of model outputs and actual yields through various regression methods¹⁰. The model outputs need not necessarily be yields; other outputs can be used instead, such as crop water consumption measured in terms of crop evapotranspiration, average soil moisture, and water excess/deficit at critical growth stages.

The problem of model tuning is linked to the sensitivity of a model. MacKerron [1992] distinguishes internal sensitivity (the sensitivity linked with the internal model variables) and external sensitivity. MacKerron and other scientists regard the tuning of internal constants as improper practice, but this author suggests that this applies only to the level of a field or farm.

In the case of regional yield forecasts, the tuning of both internal and external constants is usually necessary due to a number of methodological problems associated with the scale of the input data. Take, for instance, soil moisture storage capacity, a very important variable in semi-arid areas. The concept obviously does not hold at a district level due to the large spatial variability of soil characteristics. The same applies to planting dates, the date at which a simulation run is started, which is spread over several weeks due to different management practices of farmers, as well as to the spatial variability of weather conditions.

With a sufficient sampling effort, the large variety of actual planting dates, soil water storage capacity, etc. could be assessed, and the model run for each of the situations thus identified. Unfortunately, many input data are simply not available at a sufficient level of detail, and this includes most of the weather parameters, which are measured at meteorological stations only for some of them, sometimes at the farm level for the most important or the most readily measurable ones (rainfall, temperature).

There is an intermediate scale between point-data (P-data, i.e. those measured at points) and area-wide agricultural statistics (A-data): satellite information, the “native” scale of which is the pixel (an elementary area), variable according to the satellite, which is the smallest area the satellite sensors can actually resolve. Pixel sizes actually used in ACYF typically vary between 1 and 10 kilometers.

The issue of the inter-compatibility and inter-comparability of point data versus area-wide data is very frequently debated in the ambit of ACYF. For a more detailed discussion of some of the methodological points, the reader is referred to Hough, Gomme and Keane [1998]. The differences between point- and area-data are by no means so clear cut as might appear at first look. Take, for

⁹ Most of this discussion applies equally to descriptive, regression and simulation methods.

¹⁰ Actual yields are normally de-trended, or time is built in as a regression variable, which leads to essentially comparable results. Both cross-sectional and time series data can be used for the calibration, provided the areas covered are comparable from an agronomic point of view.

instance, radar rainfall. Although it covers vast contiguous areas, the spatial resolution is so fine that it comes close to rain gauge readings over short time periods¹¹.

In practice, in order to carry out meaningful ACYF, it is necessary to “convert” all the inputs to the same spatial scale. This is done by superposing on the area being investigated a virtual grid (similar to the grid of longitudes and latitudes) fine enough to accommodate the scales of all the parameters under consideration. The mesh size is usually in the range from 1 to 50 kilometers. A variety of techniques is now available to estimate through interpolation the climatic and other data at the grid points [Bogaert, Mahau and Beckers 1995]. This is known as gridding, a technique closely related to area-averaging, i.e. the estimation of the average value of a parameter over a given area. The estimation of average crop yield over a district is obviously an example of area-averaging, which can easily be computed by averaging the corresponding grid point values.

The issue of the spatial resolution is also intimately linked to the time resolution, not only because the weather variables become less variable over space when longer time periods are considered¹² (which is one argument in favor of longer model time steps), but also because some internal and external variables used in crop yield modeling make little sense at the finest resolutions.

“In theory, all the input parameters of a crop simulation model should have an actual physical meaning, but this cannot be achieved in practice, because the real complexity of the spatial distribution of many variables (fractal structure) cannot be apprehended by ordinary maps at any scale and because, for many variables (WHC, soil Water Holding Capacity), it simply does not make sense to treat them like A-data. There may be a need to develop concepts like ‘effective soil moisture’, the A-data equivalent of the purely P-type soil moisture. In the same way as ‘effective rainfall’ is different from actually measured rainfall, there may be a way to define an ‘effective WHC’ which, together with average area rainfall, average area ETP, ‘effective planting date’, etc. would result in meaningful average area yield estimates. The same applies to phenology: it is sometimes more useful (convenient) to use a computed indicator than observed values [Robertson 1968].” [Hough, Gommès & Keane 1998]

ACYF can be implemented following two fundamentally different approaches, which could be called gridding-then-modeling ($G \rightarrow M$) and modeling-then-gridding ($M \rightarrow G$). The data processing load is obviously larger in the $G \rightarrow M$ approach, but this no longer constitutes a problem. The main advantage of the approach is that fine resolution data can be made use of whenever available, but at the expense of implicitly using area-analogons for P-data, without knowing if the area-analogons and their processing do actually make physical and physiological sense. In addition, the fashion of very fine grids often leads to dubious interpolated values for such parameters as wind-speed or air moisture.

In the second approach ($M \rightarrow G$), modeling takes place only at the very locations from which actual data are available, resulting in faster computations with more reliable data and more trustworthy constants. The next step, however, is significantly more difficult as the resulting P-yields must now be gridded

¹¹ There is also the problem of stations having to be moved to other locations.

¹² This can be expressed in a number of ways. Individual showers sometimes bring very variable amounts of rain to close locations. When considered over longer periods (weeks, months), the differences are considerably smaller. For annual and seasonal rainfall, excellent correlations can be found between stations hundreds of kilometers apart.

and area-averaged. FAO has been using a combination of ground data and satellite indices (NDVI³) to obtain realistic area-averages in semi-arid areas [Gommes 1993, 1996].

5. To conclude...

Agrometeorological Crop Yield Forecasting (ACYF) approaches differ widely in terms of input data requirements and model complexity, resulting in very variable robustness, development and implementation costs. The main advantage of ACYF methods is that they do provide objective estimates of regional yields in advance of the harvest proper, and at the fraction of the cost of sampling techniques. As such, they constitute essential decision-making tools.

“Robustness” appears to be the main practical criterion under most circumstances, more so than accuracy. In fact, a method that performs well (accurately) under average conditions is easy to develop, but not very useful.

It is observed again and again that sophisticated crop simulation systems fail to forecast a yield increase or a drop due to a period of dry and sunny weather at an unusual time of the crop cycle, or other rather “obvious” adverse factors such as excessively wet spring conditions. It is suggested that this failure is a side effect of the complexity of the models with many components and many inputs; the forecasting method becomes too insensitive to individual inputs, particularly when the inputs have undergone very intensive pre-processing, as examined in the section on “Some Methodological Questions and Options”. The section also raised some doubts about the actual meaning of many eco-physiological parameters used in crop simulation systems when applied at the regional level.

Traditional regression models with agrometeorological “independent” variables, on the other hand, often overreact due to their dependence on too limited a number of factors.

The author suggests that the best results can be obtained with relatively simple simulation models where more emphasis is given to the accurate selection of “value-added” agrometeorological variables than to the processing of large volumes of input data. In other words, better results can be achieved through a better knowledge of crop agronomy and crop-weather relations than through increased data processing power.

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¹³ NDVI, Normalized Difference Vegetation Index, is a variable which is positively correlated to green (living) biomass. The technique used to interpolate yields with NDVI is known as Satellite Enhanced Data Interpolation (SEDI) usually carried out with the IGT software. The technique was originally developed to interpolate rainfall measured in rain gauges using a satellite cloud index [Hoefsloot 1996a, b].

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