WEBINAR

Statistical Quality Control With Applications in R

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Date: 21st October, 2022
FOCUS

1. Introduction
2. Categorisation and Activities of Statistical Quality Control
3. Monitoring and Variation in SQC
4. Statistical Process Control Tools
5. Control Charts Concept
<table>
<thead>
<tr>
<th>Introduction: Statistics</th>
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<table>
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<table>
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In general, Statistics is a powerful tool used for:

- Planning
- Administration
- Monitoring/Evaluation
- Governance/Accountability
- Public debate and informed choices

- Statistics is a science of decision making
- Statistics uses data to improve the odds of making correct decisions
--- Max Porter
- Statistics is the key to technology and the language in which man reads the universe
--- Anonymous
Quality

- Quality is defined simply as *meeting the requirements of the customer*
- Fitness for purpose or use
- **Quality as value for money:** Value for money sees quality in terms of efficiency and effectiveness.
- The totality of features and characteristics of a product or service that bear on its ability to satisfy stated or implied needs
- Quality is the total composite product and service characteristics of marketing, engineering, manufacture, and maintenance through which the product and service in use will meet the expectation of the customer

Quality has become a major business strategy that organizations that successfully improve quality can increase their productivity, enhance their market penetration, and achieve greater profitability and a strong competitive advantage (Montgomery 2013).
The control methods help to decide when:

- the process is operating at a satisfactory level.
- the process level is not satisfactory and corrective action is required to prevent the manufacture of unacceptable products.
Quality Trilogy

- **Quality Trilogy** consists of **quality planning**, **quality control**, and **quality improvement**.

- **Quality Planning** provides a system that is capable of meeting quality standards.

- **Quality Control** is used to determine when corrective action is required.

- **Quality Improvement** seeks better ways of doing things.

Juran Quality Trilogy
Statistical Quality Control

Statistical quality control (SQC) involves the use of modern statistical methods for quality control and improvement.

In Statistical quality control, simple but powerful statistical techniques are used to monitor processes and products for customer satisfaction and improvement of the process.
Categorisation in Statistical Quality Control

**Descriptive statistics** are used to describe quality characteristics and relationships. This group includes the mean, standard deviation, range, and distribution of data.

**Statistical Process Control** is used to determine process stability and it involves inspecting a random sample of the output from a process and deciding whether the characteristics of the products in the sample fall within a predetermined range.

**Acceptance sampling** involves random inspection of a sample of goods. Based on the results of the sample, a decision is made as to whether a batch of goods should be accepted or rejected.
ACTIVITIES OF SQC

- The activities of SQC are encapsulated in DMAIC
  - Define
  - Measure
  - Analyze
  - Improve
  - Control
Why Monitoring in SQC

- In Statistical Quality Control, the main reason of monitoring a process is to determine the type of variation that drives a process.

- If a process is being driven by chance variation, then the process will behave in a random manner without specific pattern, and thus it is believed that a random force is operating in the process, and such process is “in-statistical control” and in “stable condition”.

- When there is an observed distortion in the process normal pattern, then an *Assignable variation* is at work.

- When such happens, then the process is no longer stable, and it is operating in “out-of-control” condition.

- When there is an out-of-control signal, the subject expert will be allowed to interpret such a signal.
The law of variation as defined in the statistical process control says that "everything varies". In other words, no two things are exactly alike.

Reducing the variation stakeholders experience is the key to quality and continuous improvement.
Variation in SQC
Influence of Variation in SQC

• Variation drives quality, cost of production, consumer’s and producer’s risk, customer’s satisfaction, and process understanding.

• The details relationship between variation and quality control variables are depicted pictorally.
The cause of distortion in a process (the assignable variation) are due to the five Ms and E (Adekeye, 2013).
Statistical process control (SPC) is for monitoring and controlling a process to ensure that it operates at its full potential.

It determines the stability and predictability of a process and can be applied to any process where the output of the product conforming to specifications can be measured.

Statistical process control (SPC) is a philosophy, a strategy, and a set of methods for ongoing improvement of systems, processes, and outcomes.
Figure 1.1 A process – SIPOC
Statistical Process Control

SPC involves:

- Determining the critical process parameters that need to be monitored
- Setting up an initial control chart and confirming that the process is in-control
- Collecting and plotting future data on the chart and interpreting the chart to determine if the process has gone out-of-control

SPC gives operators a tool to determine when a statistically significant change has taken place in the process or when a seemingly significant change is just due to chance causes.
SPC Tools

- Frequency plots/Check sheet
- Histogram
- Pareto Analysis
- Scatter Diagram/Run Chart
- Stratification
- Cause and Effect Diagram
- Control Charts
Check sheet or Tally sheet

Check sheet or Tally sheet is a generic data collection and analysis tool that can be adapted for a wide variety of purposes.

It can be used when:

(a) data can be observed and collected repeatedly by the same person or at the same location.
(b) collecting data on the frequency or patterns of events, problems, defects, defect location, defect causes, or similar issues.
(c) collecting data from a production process.

Application: A check sheet used to collect data on telephone interruptions in a company per week.

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The Cause and Effect diagram or fishbone diagram identifies many possible causes for an effect or problem. It can be used to structure a brainstorming session. It immediately sorts ideas into useful categories. It is used when identifying possible causes for a problem when a team's thinking tends to fall into ruts.

This fishbone diagram to understand the source of periodic iron contamination.
Histogram

A histogram is the most commonly used graph to show frequency distributions for a process data.

It is used when:
(a) The data are numerical
(b) Determine the shape of the data’s distribution, especially when determining whether the output of a process is distributed approximately normally
(c) Analyzing whether a process can meet the customer’s requirements
(d) Analyzing what the output from a supplier’s process looks like
(e) Seeing whether a process change has occurred from one time period to another
(f) Determining whether the outputs of two or more processes are different
(h) You wish to communicate the distribution of data quickly and easily to others
A Pareto chart is a bar graph. The lengths of the bars represent frequency and are arranged with longest bars on the left and the shortest to the right.

- The chart visually depicts which situations are more significant.
- The pareto chart is used when
  
  (a) analyzing data about the frequency of problems or causes in a process
  
  (b) there are many problems or causes and you want to focus on the most significant
  
  (c) analyzing broad causes by looking at their specific components
  
  (d) communicating with others about your data
The scatter diagram, graphs pairs of numerical data, with one variable on each axis, to look for a relationship between them. If the variables are correlated, the points will fall along a line or curve. The better the correlation, the tighter the points will hug the line.

Run chart is a tool used to test for the randomness of a process. If the process is random, then it implies there is no autocorrelation in the process.
WHEN TO USE A SCATTER DIAGRAM

- When you have paired numerical data
- When your dependent variable may have multiple values for each value of your independent variable
- When trying to determine whether the two variables are related, such as:
  - When trying to identify potential root causes of problems
  - After brainstorming causes and effects using a fishbone diagram to determine objectively whether a particular cause and effect are related
  - When determining whether two effects that appear to be related both occur with the same cause
  - When testing for autocorrelation before constructing a control chart
Stratification

• Stratification is defined as the act of sorting data, people, and objects into distinct groups or layers.
• It is a technique used in combination with other data analysis tools.
• When data from a variety of sources or categories have been lumped together, the meaning of the data can be difficult to see.
• It separates the data so that patterns can be more visible.

• Stratification may be used under the following conditions:
  (a) Before collecting data
  (b) When data come from several sources or conditions, such as shifts, days of the week, suppliers, or population groups
  (c) When data analysis may require separating different sources or conditions
Stratification

WHEN TO USE STRATIFICATION

- Before collecting data
- When data come from several sources or conditions, such as shifts, days of the week, suppliers, or population groups
- When data analysis may require separating different sources or conditions

From the graph, the following were observed:
The data from reactor 2 and reactor 3 are circled and it is clear that for those two reactors, purity decreases as iron increases. However, the data from reactor 1, the solid dots that are not circled, do not show that relationship. Something is different about reactor 1.
Control Chart

• The control chart is the major SQC tool used primarily for monitoring processes for improvement purposes. Data, either discrete or continuous, which are generated over some time can be monitored using an appropriate control chart.

• The control chart by my definition and experience over the last two decades is a “utility tool” that can be utilized by

  - Product engineers to record and analyze test data
  - Accountants to analyze costs
  - Process engineers to determine machine and process capabilities
  - Production engineers to monitor operations
  - Inspection and quality controllers to report scrap and rework, and to analyze incoming materials quality
  - HEIs managers to improve the quality of deliverables and services.
Control Chart

- The operation of a control chart is based on observing sample statistics values plotted on a control chart which are considered to be in-control if all the plotted statistic values fall within the control limits and show no out-of-control condition.

- Likewise, the operation under investigation is considered to be subject to the assignable cause of variation when the chart shows an out-of-control condition.
Uses of Control Chart

The variation or spread, produced solely by chance cause, can in general be predicted after an initial sample series has been studied. **Phase I operation**

Control charts are effective in defects prevention. This is consistent with the “Do it right the first time” philosophy.

Control charts prevent unnecessary process adjustment, which is consistent with the “if it is not broken, don’t fix it” philosophy.

The pattern of points on the control chart provides information of diagnostic value to an experience operator or engineer, and the Control limits provide information about process capability.
Control Chart

Change in Spread of Process due to Variation

UCL
Mean
LCL

Spread of Process due to common causes of variation

Larger Spread of Process due to Special causes of variation

-3σ -2σ -1σ X +1σ +2σ +3σ
Control Charts Assumptions

(i) the normality assumption

(ii) the stationarity and uncorrelated assumption
Out of control conditions

**Points outside:** Points beyond either control limits indicates that an external influence exists, that is an assignable cause is present.

**Run:** A run is indicated when:
- 2 out of 3 successive points outside 2 sigma limits
- 4 out of 5 successive points outside 1 sigma limits.
- Eight successive points fall on the same side or
- 11 of 12 successive points fall on the same side or
- 13 of 15 successive points fall on the same side

**Trend:** In some operations there is a steady progressive change in the plotted points. This is called trend and may be caused by tool wear or machine deterioration.

**Cycle:** At times the operation is affected by cycles caused by psychological chemical reasons or by daily, weekly, or seasonal effects. These make their appearance on the chart by a definite up and down pattern, with points possibly out of control at both limits
Control Chart Zones

UCL
---
ZONE A
---
ZONE B
---

Centerline
---
ZONE C
---
ZONE C
---
ZONE B
---

LCL
---
ZONE A
---

1/3 distance from Centerline to Control Limits
Types of Control Charts

Control Charts

Variable

Attribute

Constant sample size

Varying sample size

n>= 50
Control Charts for Variables

(i) \( \bar{X} \) and the R charts
(ii) \( \bar{X} \) and S charts
(iii) X and MR Charts

The \( \bar{X} \) chart is used to control the average of the process while the R or S or MR chart is used to control the general variability of the process.

\( \bar{X} \) and R charts

\[
\begin{align*}
UCL &= \bar{X} + 3\hat{\sigma}_X \\
LCL &= \bar{X} - 3\hat{\sigma}_X \\
\hat{\sigma}_X &= \frac{\bar{R}}{d_2}
\end{align*}
\]

\[
\begin{align*}
\bar{X} &= \frac{\sum_{i=1}^{n} X_i}{n} \\
\bar{R} &= \frac{\sum_{i=1}^{m} R_i}{m} \\
\bar{X} &= \frac{\sum_{j=1}^{m} X_j}{m}
\end{align*}
\]

Range (R) Chart

\[
\begin{align*}
UCL &= \bar{R} + 3\hat{\sigma}_R \\
LCL &= \bar{R} - 3\hat{\sigma}_R \\
\hat{\sigma}_R &= d_3 \frac{\bar{R}}{d_2}
\end{align*}
\]

\[
\begin{align*}
UCL &= \bar{RD}_4 \\
CL &= \bar{R} \\
LCL &= \bar{RD}_3
\end{align*}
\]
### Structure of Data for X-bar and R Charts

#### Average After-Tax Income for All Households, by Household Income Category, 1979-2007

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<th>Second Quintile</th>
<th>Middle Quintile</th>
<th>Fourth Quintile</th>
<th>Highest Quintile</th>
<th>All Quintiles</th>
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Source: Congressional Budget Office.
Control Chart for variables: $\bar{X}$ and S Control Charts

The S-chart is appropriate for monitoring of dispersion if

(i) sample sizes are greater than or equal to 6

(ii) when subgroup sizes are unequal.

If $\sigma$ is unknown, and $m$ samples were recorded per subgroup, then an estimate of the population variance will be used

$$\hat{S} = \frac{\sum_{i=1}^{m} S_i}{m}.$$  

The control limits on the corresponding $\bar{X}$ - chart are

- $\text{UCL} = \bar{X} + A_3 \hat{S}$
- $\text{CL} = \bar{X}$
- $\text{LCL} = \bar{X} + A_3 \hat{S}$

The control limits on the corresponding $S$ - chart are

- $\text{UCL} = B_4 \hat{S}$
- $\text{CL} = \hat{S}$
- $\text{LCL} = B_3 \hat{S}$
The $\bar{X}$ and $S$ control charts are relatively easy to apply in cases where the sample sizes vary.

- The weighted average approach is used in calculating the sample mean and standard deviation.

- If $n_i$ is the number of observations in the $i$th sample, then

$$
\bar{X} = \frac{\sum_{i=1}^{m} n_i \bar{X}_i}{\sum_{i=1}^{m} n_i}
$$

$$
\bar{s} = \left[ \frac{\sum_{i=1}^{m} (n_i - 1) s_i^2}{\sum_{i=1}^{m} n_i - m} \right]^{1/2}
$$
### Computation of Control Limits for Varying Sample Size

Example for Varying Sample Size

#### Inside Diameter Measurements (mm) on Automobile Engine Piston Rings

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Observations</th>
<th>$\bar{x}_i$</th>
<th>$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74.030 74.002 74.019 73.992 74.008</td>
<td>74.010</td>
<td>0.0148</td>
</tr>
<tr>
<td>2</td>
<td>73.995 73.992 74.001</td>
<td>73.996</td>
<td>0.0046</td>
</tr>
<tr>
<td>3</td>
<td>73.988 74.024 74.021 74.005 74.002</td>
<td>74.008</td>
<td>0.0147</td>
</tr>
<tr>
<td>4</td>
<td>74.002 73.996 73.903 74.015 74.009</td>
<td>74.003</td>
<td>0.0091</td>
</tr>
<tr>
<td>5</td>
<td>73.992 74.007 74.015 73.989 74.014</td>
<td>74.003</td>
<td>0.0122</td>
</tr>
<tr>
<td>6</td>
<td>74.009 73.994 73.997 73.985</td>
<td>73.996</td>
<td>0.0099</td>
</tr>
<tr>
<td>7</td>
<td>73.995 74.006 73.994 74.000</td>
<td>73.999</td>
<td>0.0055</td>
</tr>
<tr>
<td>8</td>
<td>73.985 74.003 73.993 74.015 73.988</td>
<td>73.997</td>
<td>0.0123</td>
</tr>
<tr>
<td>9</td>
<td>74.008 73.995 74.009 74.005 74.004</td>
<td>74.004</td>
<td>0.0064</td>
</tr>
<tr>
<td>10</td>
<td>73.998 74.000 73.990 74.007 73.995</td>
<td>73.998</td>
<td>0.0063</td>
</tr>
</tbody>
</table>

#### Computation of Control Limits for $\bar{x}$ and $s$ Charts with Variable Sample Size

<table>
<thead>
<tr>
<th>Sample</th>
<th>$n$</th>
<th>$\bar{x}$</th>
<th>$s$</th>
<th>$A_3$</th>
<th>$B_4$</th>
<th>$B_4$</th>
<th>LCL</th>
<th>UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>74.010</td>
<td>0.0148</td>
<td>1.427</td>
<td>73.986</td>
<td>74.016</td>
<td>2.089</td>
<td>0.022</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>73.996</td>
<td>0.0046</td>
<td>1.954</td>
<td>73.981</td>
<td>74.021</td>
<td>2.568</td>
<td>0.026</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>74.008</td>
<td>0.0147</td>
<td>1.427</td>
<td>73.986</td>
<td>74.016</td>
<td>2.089</td>
<td>0.022</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>74.003</td>
<td>0.0091</td>
<td>1.427</td>
<td>73.986</td>
<td>74.016</td>
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<td>73.986</td>
<td>74.016</td>
<td>2.089</td>
<td>0.022</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>73.996</td>
<td>0.0099</td>
<td>1.628</td>
<td>73.984</td>
<td>74.018</td>
<td>2.266</td>
<td>0.023</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>73.999</td>
<td>0.0055</td>
<td>1.628</td>
<td>73.984</td>
<td>74.018</td>
<td>2.266</td>
<td>0.023</td>
</tr>
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<td>8</td>
<td>5</td>
<td>73.997</td>
<td>0.0123</td>
<td>1.427</td>
<td>73.986</td>
<td>74.016</td>
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<td>73.984</td>
<td>74.018</td>
<td>2.266</td>
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<td>10</td>
<td>5</td>
<td>73.998</td>
<td>0.0063</td>
<td>1.427</td>
<td>73.986</td>
<td>74.016</td>
<td>2.089</td>
<td>0.022</td>
</tr>
</tbody>
</table>

#### Graphs

- [Graph (a)](image1)
- [Graph (b)](image2)
When the standard values are known. That is when $\mu$ and $\sigma$ are known and Normality assumption is true.

If the standard values are represented by $\bar{X}''$ and $\sigma''$, the control limits for the $\bar{X}$, R and S charts are:

- **For $\bar{X}$ Chart**
  
  \[
  \begin{align*}
  UCL &= \bar{X}'' + A\sigma'' \\
  CL &= \bar{X}'' \\
  LCL &= \bar{X}'' - A\sigma''
  \end{align*}
  \]

- **For R Chart**
  
  \[
  \begin{align*}
  UCL &= D_2 \sigma'' \\
  CL &= \sigma'' \\
  LCL &= D_1 \sigma''
  \end{align*}
  \]

- **For S Chart**
  
  \[
  \begin{align*}
  UCL &= B_6 \sigma \\
  CL &= C_4 \sigma \\
  LCL &= B_5 \sigma
  \end{align*}
  \]
Control Chart For Individual Observation
(X and MR Charts)

• The X and Moving Range (MR) chart is used when

(i) Observation are expensive to get (e.g. destructive test).

(ii) Outputs are too homogeneous over short time intervals (e.g. PH of a Solution).

(iii) The production rate is slow and the interval between successive observation is long.

The control limits for the X chart are:

\[
\begin{align*}
UCL &= \bar{X} + 2.66\overline{MR} \\
CL &= \bar{X} \\
LCL &= \bar{X} - 2.66\overline{MR}
\end{align*}
\]

The control limits for the MR chart are:

\[
\begin{align*}
UCL &= 3.27 \overline{MR} \\
LCL &= 0
\end{align*}
\]

\[
\overline{MR} = \frac{\sum_{i=1}^{m} MR_i}{m - 1} \quad \bar{X} = \frac{\sum_{i=1}^{m} X_i}{m}
\]
CONTROL CHARTS FOR ATTRIBUTE

- When using attribute data, subgroup size should be large, for instance $n = 50, 100$ or even larger.
- The subgroup size may vary from subgroup to subgroup.

- There are four basic attribute control charts, these are:
  - Number of defective chart (np – Chart)
  - Fraction defective chart (p-Chart)
  - Number of defect chart (C – Chart)
  - Number of defects per unit chart (U-Chart)
Control Charts for Attribute Characteristics

<table>
<thead>
<tr>
<th>What is measured</th>
<th>Chart name</th>
<th>Attribute charted</th>
<th>Centre-line</th>
<th>Warning lines</th>
<th>Action or control lines</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of defectives in sample</td>
<td>‘np’ chart</td>
<td>np – number of</td>
<td>np</td>
<td>np ± 2√(np(1-p))</td>
<td>np ± 3√(np(1-p))</td>
<td>n = sample size</td>
</tr>
<tr>
<td>of constant size n</td>
<td>or ‘pn’ chart</td>
<td>defective in sample of size n</td>
<td>np</td>
<td>np ± 2√(np(1-p))</td>
<td>np ± 3√(np(1-p))</td>
<td>p = proportion defective</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>p ± 2√(p(1-p))</td>
<td>^ p ± 3√(p(1-p))</td>
<td>p̄ = average of p</td>
</tr>
<tr>
<td>Proportion</td>
<td>‘p’ chart</td>
<td>p – the ratio of</td>
<td>p̄</td>
<td></td>
<td>p̄ ± 3√(p̄(1-p̄))</td>
<td>n̄ = average sample size</td>
</tr>
<tr>
<td>defective in a sample of variable size</td>
<td></td>
<td>defective to sample size</td>
<td>p̄</td>
<td></td>
<td>p̄ ± 3√(p̄(1-p̄))</td>
<td>p̄ = average value of p</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>p̄ ± 2√(p̄(1-p̄))</td>
<td>p̄ ± 3√(p̄(1-p̄))</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of defects/flaws in sample</td>
<td>‘c’ chart</td>
<td>c – number of</td>
<td>c̄</td>
<td></td>
<td>c̄ ± 3√(c̄)</td>
<td>c̄ = average number of defects/flaws in sample of constant size</td>
</tr>
<tr>
<td>of constant size</td>
<td></td>
<td>defective/flaws in sample of constant size</td>
<td>c̄</td>
<td></td>
<td>c̄ ± 3√(c̄)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average number of flaws/defects in</td>
<td>‘u’ chart</td>
<td>u – the ratio of</td>
<td>ū</td>
<td></td>
<td>ū ± 3√(ū)</td>
<td>ū = defects/flaws per sample</td>
</tr>
<tr>
<td>of variable size</td>
<td></td>
<td>defective to sample size</td>
<td>ū</td>
<td></td>
<td>ū ± 3√(ū)</td>
<td>ū = average value of u</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>n̄ = sample size</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>n̄ = average value of n</td>
</tr>
</tbody>
</table>

* Only valid when n is in zone n̄ ± 25 per cent
Control Chart for Non-normal Process

To solve the problem of deviation from normality
- Transformation of data to normality and thereafter use the normal-based control chart to monitor the process.
- Use of Robust estimator such as MAD,
- Distribution free based or non-parametric control charts.
  - The Rank-sum control chart, Turkey Control Chart, Sign Control chart and many more.
- Data Distribution based control charts
  - Lognormal Based Control chart, Exponential Based Control Chart and Geometric control chart.
TAKE HOME
Out of Control Action Plan (OCAP)

1. **Operation(s)**
2. Check – Measure, Test or Inspect
3. Graph the data and send back to the operation
4. Special Cause Signal?
   - Yes: Continue Production
   - No: OCAP – Out of Control Action Plan
5. Alarm

Flowchart:

- Variables or Attributes?
  - Variables
    - Advanced charting techniques (e.g. EWMA chart, F-Step chart, CUSUM chart, etc.)
    - $\overline{X}$ – $R$ chart
    - $\overline{X}$ – $S$ chart
  - Attributes
    - Individuals chart
    - Defects or Defectives
      - Defectives
        - Constant sample size?
      - Defects
        - Constant sample size?


thank you!